Teaching Teachers to Teach Probability

A Brief produced at the Park City International Seminar
Park City Mathematics Institute July 3-8, 2017

On a Saturday night in the city, the staff in a hospital must prepare for a busy night. They must decide how many nurses and doctors (with which specializations) they need, how much medical supplies and what type, and make many other decisions. How do they make these decisions? They must use data. The data from past Saturdays tells them what usually happens during Saturday nights; drug overdoses and alcohol related accidents, among many other life-threatening occurrences. The hospital director must consider the probabilities that the staff will need to handle different emergencies (e.g. attend a car accident or a shooting), and be ready for each. These decisions rest on an understanding of probability.

In this example, the hospital director must carefully balance the hospital's human resources and future patients' health (life) and make a decision based on a knowledge of chance. This example is taken out of thousands of others that clearly demonstrate the need for understanding probabilistic and statistical concepts in everyday situations. This understanding is necessary for everyone. Therefore, it is important that students develop an understanding of probabilistic concepts during their K-12 studies. To achieve this goal, their teachers must be well prepared for this task.

The teaching of probability to students of all ages is a challenge around the world. But it is a challenge that teachers must accept and their pre-service and in-service preparation should provide them tools that allow them to meet these challenges successfully. In this document, we discuss some of these challenges and provide several suggestions concerning a teacher's preparation. We focus on the following issues:

- the importance of teaching from an informal approach to probability to develop student intuition and the transition to a more formal, mathematized approach
- the issues of language and culture related to probability
- some common misconceptions that arise when teaching probability.

Probability is a fundamental tool for statistical reasoning, but probability is also an important subject in its own right. Probability as a branch of mathematics is an old and respected area of inquiry, with powerful and beautiful results. Probability is equally fundamental to other areas of mathematics (stochastic algorithms in numerical analysis that use random processes to improve performance, cryptology, stochastic differential equations that represent processes in which some of the variables behave "randomly" in some way, and many more) and in many of the physical (Brownian motion), natural (genetics), and social sciences (creation of social networks).

Informal and Formal Approach

Teachers need preservice experiences that support their instruction in probability from both an informal, investigative approach and a formal mathematical treatment. The transition from informal to formal is essential for students, and how and when to make the transition is an important decision for the teacher.

Some countries approach probability from a high level of sophistication including different understandings of probability (frequentist and axiomatic), a variety of probability distributions (continuous and discrete), and formal counting rules. Others limit the level of desired understanding to the minimal knowledge necessary to sufficiently support instruction in statistics, which includes independence and conditional probability, but does not consider combinatorics, specialized distributions, or more advanced theoretical results.

Teachers and future teachers should have the opportunity to discuss and analyze the role of tasks presented to students. Does the chosen task increase the students' mathematical knowledge? To improve understanding of probability concepts, teachers should be encouraged to reason and give sense to tasks that offer possibilities to reason about random events from the early grades. One of the main issues to be considered is that tasks should give students possibilities not only for computing probabilities but also to make decisions based on data. Using experiments and investigative inquiry activities to promote the emergence of intuitive probability is essential. Using historical fallacies to overcome persisting misconceptions can be helpful (Borovcnik & Kapadia, 2011).

The use of instructional technology for teaching probability can enhance student understanding. Using technology to produce conjectures and justifying conclusions about random experiences can help develop the learner's growing theoretical framework. Simulations of probabilistic events allow students to recognize patterns in the results. Formality comes when that pattern is generalized to a mathematical representation. Computer simulations can serve as a bridge for informal to formal thinking about probability.

A common path to develop the concepts and methods of probability is to begin in the early years with inquiry-based activities to develop intuition through games and experiments, refining and making the activities more sophisticated as the students pass through the grades. As the concepts are developed they can be formalized by developing the associated mathematical theory. In many countries, this is the theory related to statistics. How deeply students should go into the mathematical theory of probability depends on the policies of the educational system in which the learning takes place.

From an axiomatic perspective, it is important that the teachers distinguish clearly for their students the difference between the model and reality. What is the difference between rolling a die and the mathematically formal axiomatic view of probability (attributed to Kolmogorov, Shafer & Vovk, 2006)? Very often, this is a point of confusion for students. However, when students are presented an unfair dice, the contradiction between the real situation and the wrong (usually equi-probable) model may bring the understanding that indeed there is a difference between the model and reality.

The next step towards understanding this difference may be building an axiomatic model of the particular unfair die by rolling it many times and computing the relative frequencies. Thus, the informal approach provides a formal model that more or less closely approximates the real situation.

By developing a strong mathematical background, and supplementing this with many informal investigations, teachers can apply probability to real world models and statistical processes with understanding. Counter-intuitive examples abound in probability (the birthday problem is a good example). Teachers should be aware of the many counter-intuitive examples, and use them to deepen the students' appreciation for the subject of probability.

Teachers and future teachers should recognize some important results from research such as introducing a graphical distribution for discrete random variables can help students to reason about sampling and stabilized relative frequency distributions (Serradó, 2014). Also, teachers should understand the difficulty students have in distinguishing between dependent and independent experiments and distinguishing the three probabilities P(A|B), P(B|A) and P(A and B), due to the lack of rigor in the language often used in the statements of the problem. (López et al., 2016).

Language and Culture

It is essential for teachers to be familiar with the historical development of probability both as a formal subject in mathematics and in the local context and culture. The mother tongue or home language of the students affects how the concepts of probability can be explained and understood. Simulations to illustrate the principles of probability could be helpful, since a student can "see" the probabilistic ideas evolve rather than rely only on a teacher's explanations, which may be limited due to issues of the home language.

The words alone may not be sufficient to explain definitions related to probability, but the definitions become more easily understood when presented in a specific context that is familiar to the learner. The contexts give the learner a way to put the definitions into their own reality. But appropriate contexts vary from culture to culture. Many teachers use cards and dice as tools for teaching probability, but some cultures are unfamiliar with the proportion of queens or spades, while others find dice and cards offensive since they are related to gambling. Teachers should be prepared to find alternate methods for introducing the concepts of probability that matches the experience of their students.

This, in fact, is one of the reasons teachers must be very careful with language, because within a country and within a classroom, it may be the case that not all of the cultures have the same meaning for the words. It is the responsibility of the teacher to look for the right word or even create words to explain what is necessary. This, again, is a place where context can be essential for students to develop these meanings. Language also creates confusion when there are many meanings for words like random, sample, chance, uncertainty. The different meanings can cause confusion, particularly when they may have different interpretation in daily conversation. Words like modeling, simulation, inference and others should be brought into daily conversation at school to make sure students have the right idea.

Teacher preparation courses should prepare students for the language difficulties in the regions and schools in which the students will likely work. For example, it would be helpful for prospective teachers to have a list of words and expressions in various cultures across the country to enable them to make adjustments according to the backgrounds of their students.

In school, teachers need to recognize opportunities to discuss probability in natural and social sciences. Teachers at all levels need to be open to finding opportunities to bring chance and probability into the classroom conversation; for example, when teaching geometry, teachers might pose tasks related to geometric probability. Authentic tasks are helpful when they match the experiences of the students in the classroom. Teachers and future teachers can use authentic tasks to promote modeling activities by using project work, in order to interpret sample space and evaluate risks. The task in Figure 1 is an open problem that requires students to address the effect on a route that occurs when the probabilty of being stopped by the stoplight is changed.

Figure 1 Probability of Stopping at Light Changes Route (Serradó, 2017)

The accident took place at night and the ambulance, driving at a uniform speed of 50 Km/h, did not have to stop if a traffic light was red.

1. How would you change the route selected if the driver must stop if a traffic light was red?
2. Analyze the map and decide possible sources of risk.
3. Which is the best of the multiple possible routes to move injured people from the car accident location to the hospital considering the information:
4. The uniform speed is 50 Km/h and if the probability of
   stopping at a traffic light is 1/4;

When this problem was used in a study with students, the researchers commented, “When we introduced the numerical communication in terms of risk appearing from question 4, we observed differences in the strategies used by the students. Those students with a faulty use of probability who understood the risk numerically…, developed numerical strategies to compute it and decide in accordance with their computations" (Serradó, 2017). These students were advanced in probabilistic reasoning.

Professional development needs to be careful of possible misconceptions such as identifying causality and correlation. To overcome this problem, teachers need examples: (a) To analyze untruths when reading graphs using a frequentist perspective and a theoretical perspective (b) To analyze "real problems" by simulation in order to compare probability distributions.

<u>Example of a task</u> "*Air Zland has found that on average 2.9% of the passengers that have booked tickets on its main domestic routes fail to show up for departure. It responds by overbooking flights. The Airbus A320, used on these routes, has 171 seats. How many extra tickets can Air Zland sell without upsetting passengers who do show up at the terminal too late?"* (Kaniuk, 2013; Serradó, Meletiou, & Paparistodemou, 2017)

Teachers must recognize that this task on airline tickets would be culturally inappropriate for students for whom air travel is not common. The task can be directed focused on probability if an additional question is added. For example, "*If Air Zland has sold the maximum number of tickets, what is the probability that you will be "bumped*" (not have a seat)?

Misconceptions

There are many sources of misconceptions in probability. Some misconceptions arise from differences in language and from differences in culture; others arise from the mathematical subtleties of the subject of probability. In axiomatic probability, students commonly confuse the mathematical model of a situation with the real situation. Teachers need to be aware of the common misconceptions of their students, which may be culturally or linguistically determined.

Teachers must prepare to address student misconceptions such as:

Difficulties with the idea of random − When picking a lottery number, why not choose 12345 or 00000? Is it better to keep the same number each week or to change the number each week? The misunderstanding that a number with mixed digits (72519 for example) is more "random" than 12345 is a common misconception. This (and many other difficulties) arise from misunderstanding the definition of *random* (often students are not given a formal definition at all).

Difficulties with the direction of conditional probability − Conditional probability relates the probability of one event given the occurrence of a second even. Confusing the probability of event A given event B with the probability of event B given event A is seen repeatedly in statistics when interpreting *p*-values. It is a challenge to teach students that the probability of the hypothesis given the statistic is not the same as the probability of the statistic given the hypothesis.

Difficulties with inference settings – The well-known gambler's fallacy illustrates this issue. If a coin is flipped eight times and eight heads in a row are seen, what is the probability of a tails on the next flip? The common misconception is that, in the long run we should see 50% tails, so a tail is more likely since tails need to "catch up". To dispel this misconception, texts sometimes give an extreme example (8 heads in a row) and require the students to say that we see tails on the next flip with probability ½. The mistaken belief is that if an event has happened more frequently than expected, it will happen less frequently in the future. However, later in the text when inference is considered, the student is expected to give a very different answer. If the coin is a fair coin, we would see eight heads in a row with probability $\left(\frac{1}{2}\right)^8 = \frac{1}{256} < 0.004$. The student would reject the hypothesis that the coin is fair and believe that heads predominate and a tail is less likely to appear. This example illustrates the care that must be taken in choosing examples, or ones that seem to be good for one purpose may cloud or contradict the idea in another situation.

Difficulties with probabilistic intuition − Probabilistic intuition can be challenging. At times our natural intuition is faulty, but the intuition of all students can be developed through physical experiences with and intellectual engagement in the principles of probability. In the example below, Option 2 was selected by about 85% of college students even though this option is contained in Option 1.

> *Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.*
>
> Which is more probable?
> 1. Linda is a bank teller
> 2. Linda is a bank teller and is active in the feminist movement.

Difficulties with the concept of density and unequal probabilities – The initial introduction to probability typically uses events with equal probability. The misconception that all events are equally probable stays with students, and as a consequence they struggle when moving from simple situations in which all outcomes are equally likely to situations in which events are not equally likely. Teachers should be prepared to assist students in making this transition. Understanding density is a key aspect for constructing the notion of distribution, but students commonly think that all distributions are uniform.

A good reference for such misconceptions is Fischbein, Efraim and Ditza Schnarch, (1997). The evolution with age of probabilistic, intuitively based misconceptions, *Journal for Research in Mathematics Education*, 28(1) pp. 96-105.
URL: http://www.jstor.org/stable/749665

Four Perspectives on Teacher Preparation

Preparing to Teach Probability in the Czech Republic

In the Czech Republic, the educational process in probability and statistics of a prospective teacher of mathematics at a university level may look as follows:

The bachelor student undertakes two formal courses in probability and statistics. Probability covers the following list of items: basic combinatorial concepts, Kolmogorov's probabilistic space, properties of basic probability concepts, conditional probability, types of random variables (discrete and continuous), characteristics of random variables (mean value, spread etc.), alternative distributions, binomial and hypergeometric distributions, Poisson processes, normal distributions, applications of normal distributions, and the central limit theorem. Statistics covers: probability distributions used in statistics, basic statistical concepts, selections from normal distributions, exponential distributions, point estimates (maximum credibility, impartial estimates), point estimate properties (consistency), confidence intervals (normal distribution parameters, parameters of other distributions), hypothesis testing (principles, F-test and t-test), hypothesis testing (tests on non-normal distribution parameters, chi-square test and other nonparametric tests), linear regression - general model, linear regression – applications, correlation. Both courses are taught in a formal way by providing definitions, theorems and their proofs.

The master student must take a course in mathematical education that also includes in its syllabus strategies for teaching probability.

In the official K-13 curriculum probability does not have much space. From the 10^{th} to the 13^{th} grade it is obligatory to teach the following themes:

1. Combinatorics – elementary combinatorial tasks, variations, permutations and combinations, binomial theorem, Pascal's triangle.
2. Probability - random event (experiment) and its probability, probability of the union and intersection of events, independence of events.
3. Working with data - data analysis, mean value, median, modus, quantile, spread and standard deviation.

However, each school has an obligation to make its own School Educational Program (an official document), which must include the above three items but may contain much more. Moreover, the official philosophy is that the taught topics are not the goals of the educational process; they are the tools to reach the following key competencies:

- Learning competencies
- Problem solving competencies
- Communicative competencies
- Social and personal competencies

- Civil competencies
- Working competencies

In the School Educational Program, it must be specified which competencies are being developed by teaching a specific topic.

Unlike other mathematical topics, probability (together with statistics) has the potential to develop all the above mentioned key competencies. The prospective teachers seem to be well prepared for fulfilling the K-13 curriculum requirements (in the case of probability). However, it is questionable whether they are able to develop the key competencies in their students after completing their university studies.

A Prospective on the Preparation of Teachers to Teach Probability in Guatemala

To insure our teachers are prepared to teach probability in statistics, the following program of education is required. In general, the teachers of mathematics are prepared along two paths. One path is learning the mathematical concepts and basic knowledge of probability (the content knowledge). The second path includes the instructional methodologies required to teach the content knowledge successfully. The pre-service teachers must be prepared in the uses of mathematics and be able to apply the appropriate mathematical tools when necessary. They must consider the kind of problems their students will face both in school and after they leave school. Teachers must be able to help students transition from intuitive knowledge to a formal knowledge.

Statistics and probability have become very important topics in mathematics due to their relevance in almost every human endeavor. Teachers must be trained to develop the ability to look for the applications of probability in the lives of their students. They must learn to make decisions based on all available data. They need to learn to investigate, to relate variables, to decide about samples, and to use and understand random numbers so they can teach these methods to their students. They must be trained to construct models that get as close as possible to reality, to apply the mathematics needed (the formal part of it) and the interpretations about the phenomena being studied and to make the right decisions based on their data. Teachers must have their students make measurements whenever possible.

Practice is critical. Students should spend time in practice until the concepts and practices become comfortable. Practice is essential to make the necessary inferences from the solutions students find when solving problems. Students also must learn to write properly about what they found in their studies and to share their information. The students should also write about what they infer, what they discover, and their doubts.

The teachers must know their mathematics and teach how to use the mathematics. Terms and concepts like random, sample, variation, mean, standard deviation, variables, possibilities, probability, inference, data and some others should be words of daily use. The teacher should be aware of how probability can occur in the world and in life. Technology is a strong tool to build models that represent reality, so teachers must know how to make the best use of the technology in their teaching.

Some Ideas about Teaching Probability and Training Teachers in Spain

The Spanish curricular law, still anchored in a traditional conception of learning accumulation, organizes the curriculum in five blocks: processes, methods and attitudes, numbers and algebra, geometry, functions (analysis), statistics and probability. Statistics and probability are included in the same block; however, this does not suggest any direct relationship between them. Although mathematics is taught in almost all the trajectories, it is not the same for probability. In some levels, the curriculum dictates to teach probability and in others statistics.

The curricular guidelines do not establish connections between the ontological model of chance, the uncertainty of a situation and the mathematical model of randomness. In particular, with respect to randomness, only a model is described. Randomness is seen as a contraposition to determinism. This model makes difficult the construction of the concept of random experiment (Serradó, Azcárate, & Cardeñoso, 2006). Furthermore, Spanish curricular guidelines use exclusively the concepts of random experiment and phenomena, in contrast with the wide range of terminology described in the Common Core State Standards Initiative (CCSS, 2010) or GAISE report (Franklin et al., 2007).

The 2015 guidelines for Spanish schools suggest the introduction of probability beginning in middle school, with concepts such as: elemental event, equiprobable and non-equiprobable event, sample space. In grade 10, 11 and 12, the concepts of compound, dependent and independent events are introduced. And the Kolmogorov axiomatic approach is introduced in grades 11 and 12 in both studies “Mathematics II” and “Mathematics Applied to Social Science”. There is no mention of set theory or any link in the curricular guidelines.

In Spain, the preparation of mathematics teachers who will teach probability to 12 to 18 year-old students begins with an undergraduate degree in a science-related discipline (engineering, architecture, medicine or, of course, mathematics) followed by a Master’s degree in teacher education. It is imagined that future teachers must know the curriculum centered on competencies, and professional development is also centered on four basic competences: mathematical content and epistemic analysis, citizenship competencies; didactical analysis competence, and ethical competencies. Primary teachers have a short mathematical formation including classical number, geometry, functions, and data treatment. Professional development is based on the idea of analyzing school mathematical practices in terms of epistemic quality, mediators, cognitive observations about what students learn, interactive and emotional issues and other ecological perspectives. Probability doesn’t appear in many cases. Therefore, the modeling approach proposed in the formation has no direct impact in the case of probability classes.

In the specific inter-university, Catalonia program, probability is part of the course called "Extended contents on mathematics", and a small part in other didactical courses, that focus more on algebra and geometry. In such a complement course, some interesting problems are presented, with no insistence of theoretical mathematical content. The assumption is that everyone has enough mathematical background, even if that is not completely true. Nevertheless, it has been decided that the statistics and probability notions must be included instead of more algebra and geometry. In such a course, regular misconceptions and fallacies are used to underpin basic content ideas of probability, how important it is to discuss randomness in school beginning with the early grades, and how to use modeling perspectives in school as described in our curriculum. Prospective teachers see in this course the use of Information and Communication Technologies (ICT). Opportunities for new teachers can be instigated in this course for them to learn about randomness and conditional probability, which are subjects that teachers must include in their curricular activities.

United States

In the latest publication of the *Mathematical Education of Teachers II* (2010), the recommendation for teachers at high school is a single course in Statistics and Probability. MET II noted that the Common Core State Standards include interpretation of data, an informal treatment of inference, basic probability (including conditional probability), and, in the extended standards for more advanced study, the use of probability to make decisions. In preparation for teaching this course, teachers should see real-world data sets, understand what makes a data set good or bad for answering the question at hand, appreciate the omnipresence of variability, and see the quantification and explanation of variability via statistical models that incorporate variability. For this purpose, the standard statistics course that serves future engineers and science majors in many institutions may not be appropriate. The suggested course "centers around statistical concepts and real-world case studies, and makes use of technology in an active learning environment. It would contain the following topics: formulation of statistical questions; exploration and display of univariate data sets and comparisons among multiple univariate data sets; exploration and display of bivariate categorical data (two-way tables, association) and bivariate measurement data (scatter plots, association, simple linear regression, correlation); introduction to the use of randomization and simulation in data production and inferential reasoning; inference for means and proportions and differences of means or proportions, including notions of p-value and margin of error; and introduction to probability from a relative frequency perspective, including additive and multiplicative rules, conditional probability and independence."

For teachers who plan to teach statistics, including high school courses that address the more advanced parts of the statistics standards in the CCSS or Advanced Placement courses, a second course is recommended. Topics for this course include, "regression analysis, including exponential and quadratic models; transformations of data (logs, powers); categorical data analysis, including logistic regression and chi-square tests; introduction to study design (surveys, experiments, and observational studies); randomization procedures for data production and inference; and introduction to one-way analysis of variance." Notice there is no probability in the advanced course.

The Advanced Placement program is organized by the College Board and offers exams in college level courses in many disciplines to high school students. Students that pass the exam may be able to receive college credit and placement for their high school work.
Advanced Placement (AP) Statistics is taken by more than 350,000 high school students each year and to a great extent, the AP Statistics curriculum determines the level at which probability is taught in the US. Thus, it is rare for an American high school to have a course in probability because probability is almost always studied in the context of a course in statistics.

The GAISE Report of the American Statistical Association (Franklin et al., 2007) recommends, "The concepts of probability needed for introductory statistics (with emphasis on data analysis) include relative frequency interpretations of data, probability distributions as models of populations of measurements, an introduction to the normal distribution as a model for sampling distributions, and the basic ideas of expected value and random variation. *Counting rules, most specialized distributions and the development of theorems on the mathematics of probability should be left to areas of discrete mathematics and/or calculus.* (emphasis added)" The AP Statistics course adheres to this guideline, limiting the development of probability to just that minimal amount needed in service to statistics. As a result, the probability experiences of most high school students are limited to these topics, and consequently, the preparation of teachers in the subject of probability is similarly limited in scope and depth.

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