Day 4: Genesis - Peter Gabriel = Genesis

Opener

1. We're going to start with doing the same thing, over and over. Find a closed rule for $T(n)$ satisfying

$$T(n) = 13\,T(n-1) - 30\,T(n-2)$$

with the starting data $\begin{bmatrix} 2 \\ 13 \end{bmatrix}$. If you're not sure where to begin, calculate a bit, then try taking common ratios of consecutive terms.

This problem set is brought to you by Nelly, ft. Tim McGraw.

Important Stuff

2. Calculate each of these.

a. $\left(\frac{1+\sqrt{5}}{2}\right) + \left(\frac{1-\sqrt{5}}{2}\right)$

b. $\left(\frac{1+\sqrt{5}}{2}\right) \cdot \left(\frac{1-\sqrt{5}}{2}\right)$

Sheesh, fractions *and* radicals? It's only problem 2! And it's Sunday! And it's not even 9 am yet!

3. Find two numbers with sum $salt$ and product $pepa$.

a. $salt = 13, pepa = 30$
b. $salt = 10, pepa = 21$
c. $salt = 100, pepa = -1469$
d. $salt = 1, pepa = -1$

Whatta number, whatta number, whatta mighty good number!

If you get the last one right, you're golden. Problems about Spinderella may appear later.

4. The Hsu Shay Resort features 2 ski lifts, 1 cross-country trail, and 11 downhill ski runs connecting five locations: **B**ase of Ace, **R**aindeer Crossing, **I**cy Drink, **A**ltitude Sickness, and **N**ap Zone. There are ski runs with multiple difficulty levels between some of the locations.

The resort uses tokens as a form of payment: arriving at every location via skiing or lift requires one token and buying a coffee at the Base of Ace requires one token.

a. Fill in this 5-by-5 grid showing the number of ways of traveling between any two locations, using exactly one token. There is a 1 in the upper-left corner

Who say Hsu Shay open through May? Closed on Tuesday. Hsu Shay's loose hay today, you say? New day, who pay Hsu Shay? You? They? Touché.

Cross-country skiing counts as skiing. Basically, following any arrow on the diagram costs one token.

If you're taking the chair lift to the top, drink lots of water first!

because the only way to stay at B and spend exactly one token is to buy a cup of coffee.

Useless information *you should not use* when filling in this grid: Hsu or Shay might lie to you about their ski resort, but only when they are wearing red and not blue. Also, the coffee at Base of Ace will open up your eyes, but the shop can be a little hard to find; look for the sign.

... to location

# of ways from location

	B	**R**	**I**	**A**	**N**
B	1				
R					
I					
A	1	2	0	0	1
N					

... using exactly one token.

b. Julie spends 2 tokens to go from A to B. How many different ways could she have done this?

c. Natalie spends 2 tokens to go from B to A. How many different ways could she have done this?

d. Fill in this grid showing the number of ways of traveling between any two locations, using exactly two tokens. Complete the grid using only the grid for one-token travel. Do not use the map. Do not use any techmology.

Do not use a Phone-A-Friend, for that also requires techmology. Techmology is wack!

... to location

# of ways from location

	B	**R**	**I**	**A**	**N**
B				1	
R					
I					
A	9				
N					

... using exactly two tokens.

e. Build a grid showing the number of ways of traveling between any two locations, using exactly three tokens. Complete the grid using only the other grids.

Do not pass Go, do not collect $200, do not taunt Happy Fun Ball, do not mess with the Council of Ricks.

Neat Stuff

5. Here's a recursive rule:

$$S(n) = 10\,S(n-1) - 21\,S(n-2)$$

If Arundhati uses the starting data $\begin{bmatrix} 1 \\ k \end{bmatrix}$, what values of k will produce an exponential function?

6. Find the solution to each of these systems of equations.

a. $A\begin{bmatrix} 1 \\ 7 \end{bmatrix} + B\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$

b. $A\begin{bmatrix} 1 \\ 7 \end{bmatrix} + B\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 19 \end{bmatrix}$

c. $A\begin{bmatrix} 1 \\ 7 \end{bmatrix} + B\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

d. $A\begin{bmatrix} 1 \\ 7 \end{bmatrix} + B\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} h \\ k \end{bmatrix}$

Forgot what that stuff means? We defined this shorthand previously:

$\begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$ and

$2\begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$.

7. Here's a recursive rule:

$$T(n) = 10\,T(n-1) - 21\,T(n-2)$$

Hemang uses the starting data $\begin{bmatrix} h \\ k \end{bmatrix}$. Find a closed rule for $T(n)$ if . . .

a. $h = 2, k = 10$

b. $h = 1, k = 19$

c. $h = 0, k = 1$

d. you're not given h or k

The last answer will be in terms of h and k, which are grateful to finally be used for something other than horizontal and vertical shifts.

8. Oyinka's function $J(n)$ from yesterday satisfies

$$J(n) = 7\,J(n-1) - 10\,J(n-2)$$

with starting data $\begin{bmatrix} 2 \\ 7 \end{bmatrix}$.

You saw that as n increases, the common ratio of consecutive terms approached 5. But what happens if n decreases?

a. Determine the value of $J(-1)$ that would allow the recursive rule to continue working. Specifically, $J(1) = 7\,J(0) - 10\,J(-1)$ gives this value.

b. Determine $J(-2)$ through $J(-6)$ to a reasonable number of decimal places.

c. What happens to the ratio of consecutive terms as n becomes more and more negative? The ratio is always more than 1, by the way.

Remember, the common ratio would be $J(-5)/J(-6)$, not the other way around.

9. Find closed rules for...

a. $\begin{bmatrix} 2 \\ 10 \end{bmatrix}$ for $R(n) = 10\,R(n-1) - 22\,R(n-2)$

b. $\begin{bmatrix} 2 \\ 10 \end{bmatrix}$ for $I(n) = 10\,I(n-1) - 29\,I(n-2)$

Deeper into the problem sets, things always seem to get more complex.

c. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ for $L(n) = L(n-1) + L(n-2)$

d. $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ for $Y(n) = 6\,Y(n-1) - 9\,Y(n-2)$

Oh noes.

e. $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$ for $Y(n) = 6\,Y(n-1) - 9\,Y(n-2)$

10. Research more sets of starting data or other recursions to figure out what's going on with $Y(n)$ in problem 9.

I say: hey-ey-ey-ey-ey, hey-ey-ey. I say hey! What's going on? And then I wake in the morning and I step outside, and I take a deep breath, and I get real high, and I scream from the top of my lungs: isn't this supposed to be the weekend?

Tough Stuff

11. What problem are we going to ask you to solve tomorrow? Fine, just solve it now, then.

12. The *Onemorenacci sequence* is defined by the rule

$$O(n) = O(n-1) + O(n-2) + 1$$

with starting data $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find a closed rule for the Onemorenacci sequence.

13. Prove that any positive integer can be written *in exactly one way* as the sum of one or more non-consecutive Fibonacci numbers. For example: $53 = 34 + 13 + 5 + 1$.

Hsu Shay Resort