Day 5: (NA) ^ 16 = Batman

Opener

1. We're going to start with doing the same thing, over and over.

The *Lucas sequence* is defined by

Problem 2 from yesterday's set may be helpful.

$$\begin{aligned} L(0) &= 2 \\ L(1) &= 1 \\ L(n) &= L(n-1) + L(n-2) \qquad \text{if } n > 1 \end{aligned}$$

Evaluate $L(0)$ through $L(6)$ and find a closed rule for $L(n)$.

2. We're going to start with doing the same thing, over and over.

Wait... wait... What are these numbers called again? Fibboplonki? Nibbonoochie? Tribiani? Tamagotchi? Fonzarelli? Chimichanga? Minnelli?

Problem 2 from yesterday's set may be helpful.

$$\begin{aligned} F(0) &= 0 \\ F(1) &= 1 \\ F(n) &= F(n-1) + F(n-2) \qquad \text{if } n > 1 \end{aligned}$$

Well, whatever they are, find a closed rule for them.

Important Stuff

3. Here are three pitchers' attributes of strength (STR), stamina (STM), skill (SKL), and speed (SPD).

Pitcher data provided by the Las Vegas 51s, whose slogan is "Always Bet On Grey".

	McGowan #19	Wheeler #47	Rowen #46
STR	3	4	20
STM	1	21	1
SKL	5	18	6
SPD	20	7	99

Elizabeth imagines a new pitcher, , a precise mix of these three: 20% McGowan, 35% Wheeler, and 45%

Hey Wheeler! You're a Lucas number *and* your digits are also Lucas numbers!

Rowen. Calculate the attributes of this new pitcher.

$$\text{STR} = \boxed{11} = .20 \cdot \boxed{3} + .35 \cdot \boxed{4} + .45 \cdot \boxed{}$$
$$\text{STM} = \boxed{8} = .20 \cdot \boxed{} + .35 \cdot \boxed{} + .45 \cdot \boxed{}$$
$$\text{SKL} = \boxed{} =$$
$$\text{SPD} = \boxed{} =$$

This new pitcher might answer to the name "Bzzz."

4. Are you missing your friends and family? PCMI sells care packages! They may contain bananas, raisins, ice cream bars, almonds, and napkins. There are five different kinds of care packages:

Care packages may also contain traces of meat lasagna. Do not ask for your care package to be delivered "animal style".

	Care Package				
	#1	#2	#3	#4	#5
Bananas	2	2	0	1	1
Raisins	3	0	0	3	0
Ice cream bars	5	2	1	2	1
Almonds	9	0	1	1	0
Napkins	4	0	0	3	1

Cynthia places the following order:

	# ordered
Care package #1	1
Care package #2	3
Care package #3	2
Care package #4	1
Care package #5	2

To fulfill Cynthia's order, how many of each item (bananas, raisins, etc.) need to be prepared?

Warning: ice cream bars may not be the smartest item to put in care packages. Fortunately, there are also napkins.

5. Tino and Cristina also order care packages:

	Tino's order	Cristina's order
Care package #1	0	10
Care package #2	0	30
Care package #3	0	20
Care package #4	2	10
Care package #5	0	20

To fulfill Tino's order, how many of each item (bananas, raisins, etc.) need to be prepared? Cristina's order?

6. Follow these steps to calculate this product of a matrix and a vector on an TI-Nspire CX:

A matrix is just an array of numbers. A vector is a matrix that has only one row or column.

There aren't enough TI-Nspires for everyone—please share or you can use some other technology that you're familiar with.

$$\begin{bmatrix} 3 & 4 & 20 \\ 1 & 21 & 1 \\ 5 & 18 & 6 \\ 20 & 7 & 99 \end{bmatrix} \begin{bmatrix} .20 \\ .35 \\ .45 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

a. Press [on]. If not in calculator mode, press .

b. If someone had previously been in the midst of typing in a calculation to be performed, clear it by pressing [ctrl] then [del].

Please do not push ALT in between. Also, there is no ALT key on an Nspire.

c. Press to display a menu of templates. Use the directional arrows below the screen to highlight . Press [enter] to select it.

d. A screen titled "Create a Matrix" will appear. Press [del] and [4] to change the number of rows to four. Press [tab] twice to highlight the "OK" button then press [enter] to select it.

When creating a matrix, be careful that it does not enslave humanity.

e. Fill in each of the 12 entries of the matrix by typing each one and pressing [tab] to move to the next entry. After the final entry, press [tab] to move your cursor to the right of the matrix. Do not press the multiplication key as it is not needed.

They say no one can be told what the matrix is . . . well, they're wrong, because clearly we're telling you right now.

f. Type in the vector by pressing and choosing again. This time, set the number of rows to 3 and the number of columns to 1. Fill in the numbers in that vector as before.

g. Press [enter] to calculate the product of the matrix and vector. Write the answer in the space above. What do you notice?

I noticed the instructions ended. I wonder how technology went from being wack yesterday to being good today.

7. Use technology (as you did in the previous problem) to check your work on problems 4 and 5.

8. Use technology to check your work on the Hsu Shay Resort problem from yesterday.

Sashay to Hsu Shay today, hoobae!

The Week In Review

9. Take a few minutes to look back at what you've done. List five things you learned this week, and two things you are still unsure about or would like to investigate further. We'll talk this over at the end of class.

$(NA)^8 + (HEY)^3 =$ GOODBYE

Neat Stuff

10. Here are the values of the first few numbers in the Lucas and Fibonacci sequences.

term	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$L(n)$	2	1	3	4	7	11	18	29	47	76	123	199	322	521	843
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

a. Find and describe the sums of corresponding Lucas and Fibonacci numbers.

b. Find and describe the products of corresponding Lucas and Fibonacci numbers.

c. Use the closed forms of Lucas and Fibonacci numbers to prove what you noticed about these products.

11. **a.** Something interesting happens when you take the product of two Fibonacci numbers that surround a third. What is it?

b. Something else interesting happens when you take the *sum* of two Fibonacci numbers that surround a third. What is it?

Whatizit was the horrible mascot for the Atlanta 1996 Olympics!

It's Crispin Glover's 2005 surrealist film, of course . . . which premiered in Park City!

12. The golden ratio ϕ is the number $\dfrac{1+\sqrt{5}}{2}$.

a. Show that $\phi^2 = \phi + 1$.

b. Show that $\phi^3 = \phi(\phi + 1)$ without evaluating ϕ.

c. Show that $\phi^3 = \text{blah}\phi + \text{bleh}$. You figure out the blahnks, but there's a catch: you are not allowed to write the symbol $\sqrt{5}$ anymore in this problem! Use the behavior of ϕ to guide you.

d. Show that $\phi^4 = \text{blih}\phi + \text{blöh}$, again without evaluating ϕ.

e. Show that $\phi^5 = \text{bluh}\phi + \text{blyh}$.

f. Describe a general rule for ϕ^n. Awesome!!

g. Find cool rules for ϕ^n for *negative* values of n.

The correct pronunciation of ϕ^3 is "fum".

Hm, ϕ^4 can be broken down into smaller powers of ϕ . . .

One starter is $\phi = 1 + 1/\phi$.

13. Let $f(n) = \phi^n + \phi^{-n}$. Use the results from problem 12 to evaluate $f(0)$ through $f(6)$.

$(NA)^{11} + (HEY)^1 = JUDE$

14. Calculate this expression to seven decimal places for $n = 8, 9, 10, 11$.

$$\frac{1}{\sqrt{5}} \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n$$

15. Problem 10 establishes that for Fibonacci numbers, $F(2n)$ is a multiple of $F(n)$. Find and prove a formula for this ratio:

$$\frac{F(3n)}{F(n)} =$$

16. **a.** How many ways can you pick a set of numbers from 1 to n with no consecutive numbers?

b. Solve it again with a new restriction: 1 and n are considered consecutive. Put another way: find the number of ways people could be sitting at a round table with n seats without anyone sitting next to anyone else, including the option that no one is sitting.

17. Find closed rules for...

a. $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ for $a(n) = 2\,a(n-1) - 1\,a(n-2)$

b. $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$ for $a(n) = 2\,a(n-1) - 1\,a(n-2)$

c. What's goin' on? Can you prove it?

Brother brother brother, there's far too many of you dyin' . . .

18. Find closed rules for...

a. $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ for $b(n) = 10\,b(n-1) - 25b\,(n-2)$

b. $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ for $b(n) = 10\,b(n-1) - 25b\,(n-2)$

c. $\begin{bmatrix} 1 \\ 10 \end{bmatrix}$ for $b(n) = 10\,b(n-1) - 25b\,(n-2)$

d. What's going on? Research more starting pairs.

It's weird how both SuperHuman and The Gong Show can be good at the same time. I feel like if those shows were put together, an antimatter explosion would occur.

19. Let $f(n) = A\,f(n-1) + B\,f(n-2)$ be a two-term recurrence.

$(NA)^{10}$ = KATAMARI DAMACY

a. Show that if $p(n)$ solves the recurrence, then so does $h \cdot p(n)$ for any constant h.

b. Show that if $p(n)$ and $q(n)$ each solve the recurrence, then so does $h \cdot p(n) + k \cdot q(n)$ for any constants h and k.

20. Find a closed rule for $Q(n)$.

Things get messy in episodes where Q shows up.

$$\begin{aligned} Q(0) &= 8 \\ Q(1) &= 3 \\ Q(2) &= 79 \\ Q(n) &= 19\,Q(n-2) - 30\,Q(n-3) \qquad \text{if } n > 2 \end{aligned}$$

21. Algebraically prove each of these identities. What might they be useful for, pray tell?

a. $x^n + y^n = (x+y)\left(x^{n-1} + y^{n-1}\right) - xy\left(x^{n-2} + y^{n-2}\right)$

b. $Ax^n + By^n = (x+y)\left(Ax^{n-1} + By^{n-1}\right) - xy\left(Ax^{n-2} + By^{n-2}\right)$

Tough Stuff

22. Prove this amazing fact: $L(2n)$, $\frac{F(3n)}{F(n)}$, and $(L(n))^2$ are consecutive integers. It makes you wonder if there are more examples of this sort of thing . . .

23. Given any positive integer m, which Fibonacci numbers are multiples of m?

24. What happens to the Fibonacci sequence in mod m?

For example, in mod 7 the only numbers are 0 through 6. The sequence starts 0, 1, 1, 2, 3, 5, 1, 6, 0.

a. Explain why it must be periodic and give a cap on this period in terms of m.

b. Find the period of the Fibonacci sequence for various m, looking for any patterns and conjectures.

25. Generalize problem 15 to the ratio $\frac{F(mn)}{F(n)}$ for any positive integer m.

Ask Gabe why you should be watching "The Genius"! It's really good!

26. Evaluate this sum:

$$\frac{F_0}{1} + \frac{F_1}{10^3} + \frac{F_2}{10^6} + \cdots + \frac{F_n}{10^{3n}} + \cdots$$