Day 6: Fire Burning on the ⌊Dance⌋

Opener

1. The *rhyme scheme* of a poem describes which lines of the poem rhyme with each other. For example, limericks are five-line poems with rhyme scheme AABBA. Example:

A dozen, a gross, and a score,
plus three times the square root of four,
divided by seven,
plus five times eleven,
is nine squared and not a bit more.

There once was a lovely young fellow
who loved to eat roasted marshmallow
When he turned from the fire
He felt flames getting higher
Now his buns are all toasty and yellow.

The 1st, 2nd and 5th lines rhyme and the 3rd and 4th lines rhyme, but the 1st and 3rd lines don't rhyme.

- Each line of the poem is assigned a letter so that all lines with the same letter rhyme with one another.
- The first time that a letter is used in a rhyme scheme, it must be the earliest letter in the alphabet yet to be used.

a. List four possible rhyme schemes for five-line poems, and four *invalid* rhyme schemes for five-line poems.

ABCDE is a valid rhyme scheme. Whether it's a good poem is a different question.

b. Determine the total number of possible rhyme schemes for five-line poems.

Think about how to organize your work so that you can be sure that you've found them all.

Important Stuff

2. Multiply these. Use technology such as the Nspire (see problem 6 on Day 5) whenever you want, but look for ways to save time and energy.

Save time by traveling at relativistic speeds! Save energy by traveling at zero speed.

a. $$\begin{bmatrix} 3 & 4 & 20 \\ 1 & 21 & 1 \\ 5 & 18 & 6 \\ 20 & 7 & 99 \end{bmatrix} \begin{bmatrix} .4 \\ .2 \\ .4 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

b. $$\begin{bmatrix} 3 & 4 & 20 \\ 1 & 21 & 1 \\ 5 & 18 & 6 \\ 20 & 7 & 99 \end{bmatrix} \begin{bmatrix} 40 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

c. $$\begin{bmatrix} 3 & 4 & 20 \\ 1 & 21 & 1 \\ 5 & 18 & 6 \\ 20 & 7 & 99 \end{bmatrix} \begin{bmatrix} .4 & 40 \\ .2 & 20 \\ .4 & 40 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}$$

d. $$\begin{bmatrix} 3 & 4 & 20 \\ 1 & 21 & 1 \\ 5 & 18 & 6 \\ 20 & 7 & 99 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix}$$

Save time by using a blue box that is smaller on the outside. Save energy by adding more mass!

e. $$\begin{bmatrix} 3 & 4 & 20 \\ 1 & 21 & 1 \\ 5 & 18 & 6 \\ 20 & 7 & 99 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix}$$

f. $$\begin{bmatrix} 3 & 4 & 20 \\ 1 & 21 & 1 \\ 5 & 18 & 6 \\ 20 & 7 & 99 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$$

3. Siaka knows this rule that takes points and produces new ones in the plane:

I think they met in college, or something.

$$(x, y) \mapsto (y, 6x + y)$$

a. Make a simple shape in the coordinate plane. Draw the shape that results after applying the rule. Is the new shape larger or smaller than the original?

b. A *fixed point* is a point for which (a, b) maps to itself under the rule. Determine all fixed points, if any, for this rule.

c. A *scaled point* is a point for which (a, b) maps to (ka, kb) under the rule, where k is a number. Show that $(1, 3)$ is a scaled point for this transformation.

Fixed points are scaled points, because they work when $k = 1$.

d. Find and graph all scaled points for this rule.

4. **a.** How many rhyme schemes are possible for a five-line poem whose scheme starts with AABB? ABAC?

b. Complete this table of the number of possible rhyme schemes:

It must be a table for ants! How could it possibly fit on this page!

	# of letters used					
	1	2	3	4	5	6
1-line poems	1					
2-line poems	1	1				
3-line poems	1		1			
4-line poems						
5-line poems		15				
6-line poems						

I still say only ants could fit in that table.

c. Look for patterns in the table that can be justified using what you know about rhyme schemes.

This is not the time for closed forms, if that's what you're thinking.

Neat Stuff

5. Melissa wants to write a seven-line poem with a new rhyme scheme each day. How long can she last without repeating any rhyme scheme?

6. Jace knows another rule that takes points and produces new ones: $(x, y) \mapsto (y, x+y)$. Triangle FLO has points $F(1,1), L(2,1), O(0,0)$.

They met on a disastrous Tinder date, but stayed friends.

a. Graph triangle FLO. Graph the shape that results after applying the rule. Is the new shape larger or smaller than the original?

Are you feeling the FLO? Or saving money on car insurance? Or rounding down?

b. Graph the shape that results after applying the rule a second time.

c. . . . a third time . . . a fourth time.

d. What's happening? Any thoughts about why this is happening?

7. Two days ago Table 11 suggested finding a three-term recurrence relation

The T stands for Table 11. T also stands for Terry Tao, and for Tschebysheff!

$$T(n) = A\,T(n-1) + B\,T(n-2) + C\,T(n-3)$$

satisfied by the sequence $T(n) = 2^n + 3^n + 5^n$. Try it!

8. It's a terrible day at Hsu Shay: only Altitude Sickness and the Base of Ace are open. Any path that involves the other locations is out of order.

Mark has ten tokens to spend. How many different ways can he spend all ten tokens?

To clarify, "coffee coffee up down" and "up down coffee coffee" are different.

9. Start with the point $(-8, 5)$ and follow the recursion $(x, y) \mapsto (y, x+y)$ for a while. Plot all the points you find in this way. Describe the path taken by these points, and the path taken by points that come *before* $(-8, 5)$ under the same recursion.

What point (x, y) maps to $(-8, 5)$?

10. Find a closed rule for $Q(n)$ given

$$Q(n) = 19\, Q(n-2) - 30\, Q(n-3)$$

and the starting data $Q(0) = 8, Q(1) = 3, Q(2) = 79$.

Huh. Feels like there should be a $Q(n-1)$ poking around here somewhere.

11. Say, here's an interesting rule:

$$Q(n) = 3\, Q(n-1) - 3\, Q(n-2) + Q(n-3)$$

a. Use the starting data $Q(0) = 0, Q(1) = 1, Q(2) = 4$. What happens?

b. Try a different set of starting data and graph $Q(n)$. What do you notice?

c. . . . what in the heck is going on here?

Nothing happens, not even if you type XYZZY. You are in a maze of twisty little problems, all different.

Tough Stuff

12. The *Twomorenacci sequence* is defined by the rule

$$T(n) = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ T(n-1) + T(n-2) + 2, & n > 1 \end{cases}$$

Find a closed rule for the Twomorenacci sequence.

Is that a twomore? *It's not a twomore!!* (Best said in an Austrian accent.)

13. Experiment with the recurrence relation

$$C(n) = 2x \cdot C(n-1) - C(n-2)$$

and the starting data $C(0) = 1, C(1) = x$.

The C stands for craziness. C also stands for Chebyshev!