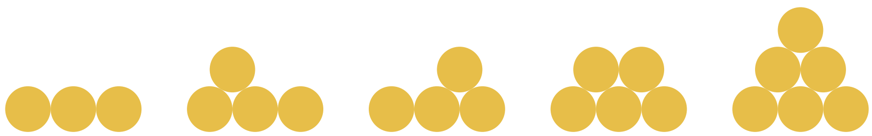
Day 7: ⌈Dancing⌉

Opener

1. Danny lives in two-dimensional Flatland. He's stacking coins on top of each other. Here are all five ways he can stack coins on top of each other, with 3 coins on the bottom row.

In Flatland, apartments are called "lorries".



a. How many different ways can Danny create stacks of coins if he starts with a row of four coins on the bottom?

b. Complete this table giving the number of ways Danny can create stacks of coins, based on the number of coins in the bottom row.

We can't say enough about how shockingly difficult it is for Danny to build these structures, living in two dimensions. He can't even see most of them when they're finished. So be careful when you describe his work as "plane".

# of coins in bottom row	# of possible coin stacks
1	
2	
3	5
4	
5	

Important Stuff

2. **a.** Using the one-token grid from problem 4a on Day 4, explain why there are exactly 5 ways to go from Icy Drink to Base of Ace using exactly two tokens.

b. Think of the grids from the Hsu Shay Resort problem (Day 4) as 5-by-5 matrices. What happens when you square the matrix represented by the one-token grid? What happens when you cube the matrix represented by the one-token grid? Verify your conjectures (with or without technology).

The operators of Hsu Shay definitely have a profit motive! They say that all that Shay wants is another token, he's gone tomorrow, boy.

3. Here are the matrices from Day 4 showing the number of ways a person could travel between locations of the Hsu Shay Resort using one, two, or three tokens.

How can we be expected to follow these matrices . . . if they can't even fit inside the page??

$$\begin{bmatrix} 2 & 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 3 & 0 \\ 5 & 2 & 1 & 2 & 1 \\ 9 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 3 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 \\ 1 & 2 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 11 & 2 & 1 & 2 & 1 \\ 6 & 6 & 0 & 3 & 3 \\ 17 & 4 & 1 & 6 & 3 \\ 12 & 2 & 0 & 10 & 2 \\ 9 & 6 & 1 & 4 & 3 \end{bmatrix}$$

two-token grid — one-token grid — three-token grid

The second column of the three-token grid is double the fourth column of the two-token grid.

a. Using the resort map, explain why this happens.
b. Using the matrices, explain why this happens.
c. Look for some other relationships of this kind.

The second column of the matrix represents Raindeer Crossing. Or maybe Raisins.

4. **a.** Describe the result of multiplying this matrix and vector.

$$\begin{bmatrix} 2 & 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 3 & 0 \\ 5 & 2 & 1 & 2 & 1 \\ 9 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

The result is tangy, with an oaky finish. Also the result is a vector.

b. Describe the result of multiplying these matrices.

$$\begin{bmatrix} 2 & 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 3 & 0 \\ 5 & 2 & 1 & 2 & 1 \\ 9 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

This time the result is a combination of many factors, most of them linear, with a silky smooth rectangular exterior associated with blue and red pills.

5. **a.** Draw a triangle with points $S(0.2, 2), A(1, 2), M(3, 6)$. Determine the area of the triangle.
b. Transform *SAM* according to the rule

$$(x, y) \mapsto (y, -10x + 7y)$$

Find the coordinates of the three new points.
c. Draw the new triangle and compute its area. How does the new area compare to the original?
d. Compute this multiplication. What do you notice?

$$\begin{bmatrix} 0 & 1 \\ -10 & 7 \end{bmatrix} \begin{bmatrix} 0.2 & 1 & 3 \\ 2 & 2 & 6 \end{bmatrix}$$

You'll have to decide whether Sam is an Autobot or a Decepticon or perhaps just a LaBeouf. As long as he makes the transforming noise, does it really matter?

Neat Stuff

6. Crhyme schemes are like regular rhyme schemes, with one additional rule.

- Each line of the poem is assigned a letter so that all lines with the same letter rhyme with one another.
- The first time that a letter is used in a Crhyme scheme, it must be the earliest letter in the alphabet yet to be used.
- *No letter can be more than one higher (in the alphabet) than the letter immediately before it.*

just like in high school
not everything rhymes
including this poem
with crhyme scheme ABCD

Every Crhyme scheme is also a valid rhyme scheme, but not necessarily vice versa. For example, ABCAC is a rhyme scheme, but it isn't a valid Crhyme scheme.

all i know is
i would have paid more attention
in english class
if there were more stuff
about crime schemes

a. How many four-line Crhyme schemes are there?

b. Complete this chart of the number of valid Crhyme schemes.

	Last letter in the scheme						
	A	B	C	D	E	F	total
1-line poems	1						
2-line poems	1	1					
3-line poems	2		1				5
4-line poems							
5-line poems	14						
6-line poems							

7. Marissa thinks there may be a way to use resort-like maps and matrices to figure out the number of Crhyme schemes. What do you think? Let's try it!

8. Linda wants to write a seven-line poem with a new Crhyme scheme each day. How long can she last without repeating?

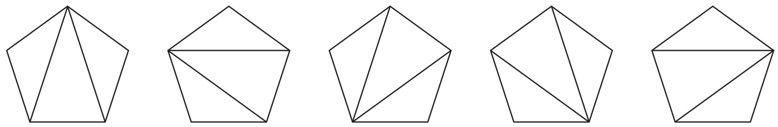
today my father bought a
dvd of law and ordah . . .

9. Establish a one-to-one correspondence between each of Danny's coin stacks (with a bottom row of n coins) and a Crhyme scheme for a n-line poem.

Pinball tip of the day: tilting Danny's coin stacks 30 degrees clockwise may offer some insight, but will also end your ball and you will receive no bonus.

10. Here are all the ways that a regular pentagon can be cut into 3 triangles by drawing two non-intersecting diagonals.

These ways recently became the hosts of some show on Fox News.



Count the number of ways that a regular $(n+2)$-gon can be cut into n triangles by drawing $(n-1)$ non-intersecting diagonals. Hint: Once you get to the regular hexagon, don't forget to include the triangulations that create an equilateral triangle within the hexagon.

11. The recursive rule $(x, y) \mapsto (y, -21x+10y)$ takes a point and produces a new point. Repeating this recursion gives a sequence of points, as long as you have starting data For each set of starting data, find the next three points in the sequence.

Beware, using the point $(4, 20)$ may result in classroom snickering.

a. $(2, 10)$
b. $(4, 20)$
c. $(\frac{10}{21}, 2)$
d. $(1, 3)$
e. $(a, 3a)$
f. $(1, 7)$

12. **a.** Build some other polygons and transform them according to the rule in problem 5. What happens to the shape of the polygons? What happens to the area of the polygons?
b. Find all the *scaled points* for this recursion, points (a, b) taken to (ka, kb) for some number k.

The sound effect is a little like "wuh-wuh-wah-whah" but there are many variations.

13. In a circle with diameter 100, a chord of length 2 is drawn. Point P is the midpoint of the chord. What is the shortest distance from P to the circle's circumference? Give your answer as a decimal to 26 places. Wait, *what*?

Really, 26 decimal places?? Why would you want me to . . . oh my gravy what is going on here

Tough Stuff

14. Explain why that just happened. (The thing in problem 13.)