Day 11 (July 12, 2017)

Modified Opener: Find a closed form expression for $v(n)$ satisfying the recursive rule $v(n) = \begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix} v(n-1)$ starting with $v(0) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Part 1: Finding scale factors and "I Can" vectors.

$$\begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ ky \end{bmatrix} \longrightarrow \begin{aligned} 7x - 3y &= kx \\ x + 3y &= ky \end{aligned}$$

$$\begin{aligned} (7-k)x - 3y &= 0 \\ x &= (k-3)y \end{aligned}$$

subst

$y \neq 0$

$$(7-k)(k-3)\cancel{y} - 3\cancel{y} = 0$$

$$(7-k)(k-3) - 3 = -k^2 + 10k - 24 = -(k-6)(k-4) = 0$$

So the scale factors are $k = 4$ and $k = 6$.

For $k = 4$, we get $x = y$ so one "I Can" vector for this scale factor is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

For $k = 6$, we get $x = 3y$ so one "I Can" vector for this scale factor is $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

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$\begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$
$= 4\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Part 1: Finding scale factors and "I Can" vectors.

For $k = 4$, we get $x = y$ so one "I Can" vector for this scale factor is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

For $k = 6$, we get $x = 3y$ so one "I Can" vector for this scale factor is $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Part 2: Use the scale factors and "I Can" vectors to find $v(n)$.

eigenvalues

Express the starting vector as a combination of the "I Can" vectors:

eigenvectors

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 3 \\ 1 \end{bmatrix} \text{ so}$$

$$\begin{aligned} v(1) = \begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} &= \begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}\left(5\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) \\ &= 5\begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ &= 5 \cdot 4\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1 \cdot 6\begin{bmatrix} 3 \\ 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} v(5) = \begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}^5\begin{bmatrix} 2 \\ 4 \end{bmatrix} &= \begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}^5\left(5\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) \\ &= 5\begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}^5\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}^5\begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ &= 5 \cdot 4^5\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1 \cdot 6^5\begin{bmatrix} 3 \\ 1 \end{bmatrix} \end{aligned}$$

$$v(n) = \begin{bmatrix} 7 & -3 \\ 1 & 3 \end{bmatrix}^n\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 5 \cdot 4^n\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1 \cdot 6^n\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \cdot 4^n - 3 \cdot 6^n \\ 5 \cdot 4^n - 6^n \end{bmatrix}$$