Inquiry as a Path to Thinking Classrooms

Week 2 July 2017

PCMI Reflecting on Practice

Unit 2 Classroom Practice
Day 1 – Characteristics of a thinking classroom and how they can be enacted

"Working as a community of inquirers, students become immersed in asking questions, wrestling with constructing, exploring and explaining mathematical meanings around a task set by the teacher or a question originating from the students. The task need not necessarily be open-ended but it has to provide students with an achievable challenge, offer exploration, encourage creativity and support decision-making."

Calleja, J. (2016). Teaching mathematics through inquiry: A continuing professional development programme design. *Journal of the International Society for Design and Development in Education. 2*(9). *http://educationaldesigner.org/ed/volume3/issue9/article30/index.htm*

Video Norms Protocol

- Video clips are to spur discussion, not criticism
- Video clips are to spur inquiry, not judgement
- Video clips are to provide a snapshot of a particular moment

Be sure to cite specific examples (evidence) from the clip

Chocolate Chip Cookie Video

TIMSSVIDEO Australia 3 Data Collection and Representation http://www.timssvideo.com/33

Round Robin Protocol

Everyone at the table should make a short statement or comment related to the question or discussion; going around in consecutive order with no input or discussion from others until everyone has had a turn.

A Thinking Classroom Summary

- conducive to thinking
- occasions thinking
- thinking individuals
- individuals thinking collectively, learning together
- constructing knowledge and understanding through activity and discussion
- teacher fosters thinking
- teacher expects thinking, both implicitly and explicitly

Liljedahl, P. (in press). Building thinking classrooms: Conditions for problem solving. In P. Felmer, J. Kilpatrick, & E. Pekhonen (Eds.) Posing and Solving Mathematical Problems: Advances and New Perspectives. New York, NY: Springer. 5

Variables of Classroom Practice

1. The type of tasks used, and when and how they are used
2. The way in which tasks are given to students
3. How groups are formed, both in general and when students work on tasks
4. Student work space while they work on tasks
5. Room organization, both in general and when students work on tasks
6. How questions are answered when students are working on tasks
7. The ways in which hints and extensions are used while students work on tasks
8. When and how a teacher levels their classroom during or after tasks
9. Assessment, both in general and when students work on tasks

Liljedahl, P. (in press). Building thinking classrooms: Conditions for problem solving. In P. Felmer, J. Kilpatrick, & E. Pekhonen (Eds.) Posing and Solving Mathematical Problems: Advances and New Perspectives. New York, NY: Springer. 2-3, 6-7

Using Procedures & Making Connections Graphs

Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership*, *61*(5), 12–17.

Building a Thinking Classroom General Findings – All Nine Elements

1. The type of tasks used, and when and how they are used
2. The way in which tasks are given to students
3. How groups are formed, both in general and when students work on tasks
4. Student work space while they work on tasks
5. Room organization, both in general and when students work on tasks
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7. The ways in which hints and extensions are used while students work on tasks
8. When and how a teacher levels their classroom during or after tasks
9. Assessment, both in general and when students work on tasks

http://peterliljedahl.com/wp-content/uploads/Building-Thinking-Classrooms-Feb-14-20151.pdf

Unit 2 Classroom Practice

Day 2 – Structures & strategies that inhibit & contribute to flow, & support student engagement with feedback, hints, & extensions

"The objective in mathematics is not to obtain the highest ranking, the highest 'score,' or the highest number of prizes and awards; instead, it is to increase understanding of mathematics (both for yourself, and for your colleagues and students), and to contribute to its development and applications. For these tasks, mathematics needs all the good people it can get."

Terence Tao, Fields Medal winner, Professor of Mathematics, UCLA
https://terrytao.wordpress.com/career-advice/does-one-have-to-be-a-genius-to-do-maths/

Desmos.com
https://www.desmos.com/

Vertical Non-Permanent Surfaces

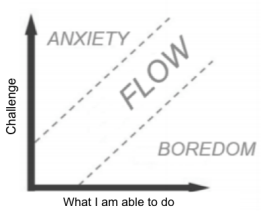
#VNPS Dry-Erase Poster 38"x58" with tinted grid to make writing neater: Rollable
Price: $29.95 + $8.50 shipping (*no shipping with Prime*) http://tinyurl.com/BuyVNPS
Or https://www.amazon.com/gp/product/B00FMYHCNW/ref=ox_sc_act_title_1?ie=UTF8&psc=1&smid=AMF4GHIYSC4SS

Consecutive Integer Task

Which positive integers can be written as the sum of two or more consecutive positive integers?

Two Classroom Scenarios

Liljedahl, P. (2016). Flow: A framework for discussing teaching. Paper Presented at PME http://www.peterliljedahl.com/wp-content/uploads/PME-2016-Flow-and-Teaching.pdf



Building a Thinking Classroom General Findings – All Nine Elements

1. The type of tasks used, and when and how they are used
2. The way in which tasks are given to students
3. How groups are formed, both in general and when students work on tasks
4. Student work space while they work on tasks
5. Room organization, both in general and when students work on tasks
6. How questions are answered when students are working on tasks
7. **The ways in which hints and extensions are used while students work on tasks**
8. When and how a teacher levels their classroom during or after tasks
9. Assessment, both in general and when students work on tasks

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Unit 2 Classroom Practice

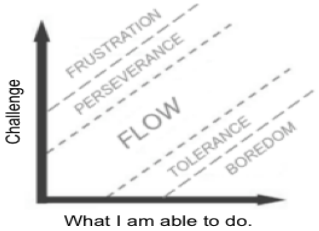
Day 3 – Productive struggle & questions/question stems that push or probe students to reveal their thinking

"To fully realize the benefits of inquiry learning, students must be encouraged to continually challenge solutions, even those that they find themselves. Questions must be weighed against logic and reason. No student should be satisfied without being able to prove that his or her solution is the correct one."

Levasseur, K. & Cuoco, A. (2003). Mathematical habits of mind. In H. L. Schoen, *Teaching mathematics through problem solving* (pp. 27-37). Reston VA: The National Council of Teachers of Mathematics Inc.

Two Classroom Scenarios

Liljedahl, P. (2016). Flow: A framework for discussing teaching. Paper Presented at PME http://www.peterliljedahl.com/wp-content/uploads/PME-2016-Flow-and-Teaching.pdf



Classroom-based Indicators of Success

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author. p. 49.

Smith, M. (2000). Reflections on practice: Redefining success in mathematics teaching and learning. *Mathematics Teaching in the Middle School*, *5*(6), 378-382, 386.

Teaching Moves that Block Productive Struggle in Math Students

Kang, H. (2016, March 31). 8 teaching habits that block productive struggle in math students inhibiting factors to productive struggle. MIND Research Institute Research blog http://blog.mindresearch.org/blog/productive-struggle-in-math

Staircase Transcript

Annenberg Learner. Teaching Math A Video Library. www.learner.org/resources/series34.html#

Push & Probe Questions Quote

The only reasons to ask questions are:

- To probe or uncover students' thinking: understand how students are thinking about the problem, discover misconceptions, use students' understanding to guide instruction
- To push or advance students' thinking: make connections, notice something significant, justify or prove students' thinking

Black, P. Harrison, C., Lee, C., Marshall, E., & Wiliam, D. (2004). Working inside the black box: Assessment for learning in the classroom. *Phi Delta Kappan*, *86*(1), 9-21.

Question Prompts and Stems to Support Mathematical Discourse

Adapted from National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Questioning Role Play

Area

Student:

- You are finding the area of a square with a side length 4.
- You are confusing finding the area of a square with finding the perimeter of a square. As such, to get your answer you count all the sides of the square and get an answer of 16.
- You are proficient in addition, subtraction, multiplication, and division.

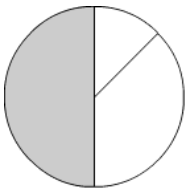
Teacher:

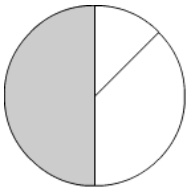
- You want to find out what answer your student got to the question, "What is the area of a square with a side length of 4?"
- Determine what conceptual understanding the student has by asking questions, especially questions that encourage elaborated responses.

Fractions

Student:

- You are trying to find what fraction of the circle is shaded.
- You believe that $\frac{1}{3}$ has been shaded because one part out of the three parts has been shaded.
- You have forgotten that each of the parts of the fraction have to be equal.
- You know that the numerator represents the number of parts and that the denominator represents the number of parts in a whole.





Teacher:

- Your student is working on finding what fraction of the circle is shaded.
- Determine what understanding the student has by asking questions, especially questions that encourage elaborate responses.

Logarithms

Student:

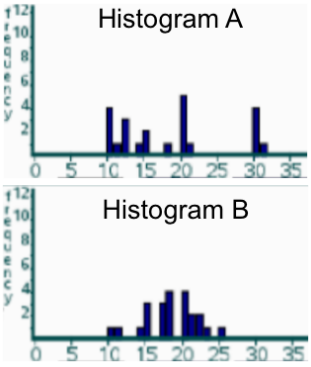
- You are in algebra 2 and have learned about logarithms.
- You are simplifying $\frac{\ln 4}{2}$.
- You get an answer of ln 2 because you think $\frac{\ln 4}{2}$ is equivalent to $\frac{\ln 4}{2} = \ln 2$.

Teacher:

- You are an Algebra 2 teacher. You want to find out what answer your student got to the question: Rewrite $\frac{\ln 4}{2}$.
- Your student is simplifying $\frac{\ln 4}{2}$ and has the correct solution. In fact, this just happens to be correct because instead $\frac{\ln 4}{2} = \frac{1}{2}\ln 4 = \ln 4^{\frac{1}{2}} = \ln 2.$
- Determine what understanding the student has by asking questions, especially questions that encourage elaborate responses.

Statistics

Student:



- You are in an introductory statistics unit and have studied measures of center (mean, median) and variability (standard deviation, interquartile range). You are trying to describe which histogram has more variability.
- You think a plot has more variability if it has more peaks.
- You say plot A has more variability.

Teacher:

- You are teaching an introductory statistics unit and have covered means, medians, standard deviation and interquartile range. Your student is trying to describe the variability of the histograms shown. You know that variability is related to how far the data values vary from either the mean or the median.
- Determine what understanding the student has by asking questions, especially questions that encourage elaborate responses.

Limits

Student:

- You are in the first few weeks of calculus, having taken a precalculus course. You are calculating the $\lim_{x\to 3} \frac{x^2-9}{x-3}$.
- You simplify $\frac{x^2-9}{x-3} = \frac{x^2}{x} + \frac{-9}{-3} = x + 3$ and find that $\lim_{x\to 3} (x + 3) = 6.$

Teacher:

- You are teaching the first unit in calculus on limits. Your student is calculating $\lim_{x\to 3} \frac{x^2-9}{x-3}$, and you want to know what answer he finds.
- Determine what understanding the student has by asking questions, especially questions that encourage elaborate responses.

Kaplinsky, R., & Canham, M. (2012, March 12). Practicing questions as a tool to assess student understanding. *CMC ComMuniCator 36*(3), pp. 28-30.

Questioning Scenarios, Robert Kaplinsky http://robertkaplinsky.com/questioning-scenarios/

Unit 2 Classroom Practice

Day 4 – Strategies for encouraging discussions in a thinking classroom

"Inquiry math is different from traditional math in that students work with partners and whole-group instruction to construct mathematical explanations that make sense to them. Students are presented with opportunities to verbally explain their thinking processes to the teacher and class, and it is this exchange of ideas that provides the foundation for true understanding of mathematical concepts."

Chapko, M. A., & Buchko, M. (2004). Math instruction for inquiring minds. Reston, VA: National Council of Teachers of Mathematics, 84, p. 33.

John's 20 oz. Filet Mignon Task

An advertisement for Clyde' states: Filet Mignon, 5-ounce $14.95; 8-ounce $19.95. John wants to have a 20 oz. filet mignon. How much should the restaurant charge him?

Levels of Discourse

- Level 0 – The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher's questions.
- Level 1 – The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.
- Level 2 – The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another's work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.
- Level 3 – The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.

Stein, C. (2007). Let's talk: Promoting mathematical discourse in the classroom. *Mathematics Teacher*, *101*(4), 285-289.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, *35*(2), 81–116.

———. (2014). Describing levels and components of a math-talk learning community. In E. A. Silver & P. A. Kenney (Eds.), *More Lessons Learned from Research Volume 1* (pp. 125-134). Reston, Va.: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author. p. 32.

Sequencing Discussions

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Penny's Marbles Task

Penny had a bag of marbles. She gave one-third of them to Rebecca and one-fourth of the remaining marbles to Aman. Penny then had 24 marbles left in her bag. How many marbles were in the bag to start with?