Motivation

My work on this paper began over 10 years ago with my research on the AHA! experience and the profound effects that these experiences have on students' beliefs and self-efficacy about mathematics (Liljedahl, 2005). That research showed that even one AHA! experience, on the heels of extended efforts at solving a problem or trying to learn some mathematics, was able to transform the way a student felt about mathematics as well as his or her ability to do mathematics. These were descriptive results. My inclination, however, was to try to find a way to make them prescriptive. The most obvious way to do this was to find a collection of problems that provided enough of a challenge that students would get stuck, and then have a solution, or solution path, appear in a flash of illumination. In hindsight, this approach was overly simplistic. Nonetheless, I implemented a number of these problems in a grade 7 (12-13 year olds) class. The teacher I was working with, Ms. Ahn, did the teaching and delivery of problems and I observed. The results were abysmal. The students did get stuck. But not, as I had hoped, after a prolonged effort at solving the problem. Instead, they gave up almost as soon as the problem was presented to them. There was some work attempted when the teacher was close by and encouraging the students, but as soon as she left the trying stopped. After three days of trying to occasion an AHA! experience in this fashion, Ms. Ahn and I agreed that we now needed to give up. But I wanted to understand what had happened, so I stayed on for a week and just watched Ms. Ahn teach her class.

After three days of observing Ms. Ahn's normal classroom routines I began see what was going on. That the students were lacking in effort was immediately obvious, but what took time to manifest was the realization that what was missing in this classroom was that the students were not thinking. More alarming was that Ms. Ahn's teaching was predicated on an assumption that the students either could not, or would not, think. The classroom norms (Yackel & Rasmussen, 2002) that had been established in Ms. Ahn's class had resulted in, what I now refer to as, a non-thinking classroom. Once I realized this I proceeded to visit other mathematics classes – first in the same school and then in other schools. In each class I saw the same basic behaviour – an assumption, implicit in the teaching, that the students either could not, or would not think. Under such conditions it was unreasonable to expect that students were going to spontaneously engage in problem solving enough to get stuck, and then persist through being stuck enough to have an AHA! experience.

What was missing for these students, and their teachers, was a central focus in mathematics on thinking. The realization that this was absent in so many classrooms that I visited motivated me to find a way to build, within these same classrooms, a culture of thinking, both for the student and the teachers. I wanted to build, what I now call, a *thinking classroom* – a classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together, and constructing knowledge and understanding through activity and discussion.

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General Methodology

The research to find the elements and teaching practices that foster, sustain, and impeded thinking classrooms has been going on for over ten years. Using a framework of noticing (Mason, 2002)[1], I initially explored my own teaching, as well as the practices of more than forty classroom mathematics teachers. From this emerged a set of nine elements that permeate mathematics classroom practice – elements that account for most of whether or not a classroom is a thinking or a non-thinking classroom. These nine elements of mathematics teaching became the focus of my research. They are:

1 At the time I was only informed by Mason (2002), Since then I have been informed by an increasing body of literature on noticing (Fernandez, Llinares, & Valls, 2012; Jacobs, Lamb, & Philipp, 2010; Mason, 2011; Sherin, Jacobs, & Philipp, 2011; van Es, 2011).

2 Levelling (Schoenfeld, 1985) is a term given to the act of closing of, or interrupting, students' work on tasks for the purposes of bringing the whole of the class (usually) up to certain level of understanding. It is most commonly seen when a teacher ends students work on a task by showing how to solve the task.

1. the type of tasks used, and when and how they are used;
2. the way in which tasks are given to students;
3. how groups are formed, both in general and when students work on tasks;
4. student work space while they work on tasks;
5. room organization, both in general and when students work on tasks;
6. how questions are answered when students are working on tasks;
7. the ways in which hints and extensions are used while students work on tasks;
8. when and how a teacher levels[2] their classroom during or after tasks;
9. and assessment, both in general and when students work on tasks.

Ms. Ahn's class, for example, was one in which:

1. practice tasks were given after she had done a number of worked examples;
2. students either copied these from the textbook or from a question written on the board;
3. students had the option to self-group to work on the homework assignment when the lesson portion of the class was done;
4. students worked at their desks writing in their notebooks;
5. students sat in rows with the students' desk facing the board at the front of the classroom;
6. students who struggled were helped individually through the solution process, either part way or all the way;
7. there were no hints, only answers, and an extension was merely the next practice question on the list;
8. when "enough time" time had passed Ms. Ahn would demonstrate the solution on the board, sometimes calling on "the class" to tell her how to proceed;
9. and assessment was always through individual quizzes and test.

Liljedahl, P. (in press). Building thinking classrooms: Conditions for problem solving. In P. Felmer, J. Kilpatrick, & E. Pekhonen (eds.) Posing and Solving Mathematical Problems: Advances and New Perspectives. New York, NY: Springer. (pages 2-3; 5-6)