Day 3: Greatest Carnivorous Dinosaur

Opener

1. Westley takes a 200 mm-by-88 mm rectangle and starts cutting! He cuts away the largest possible *square* he can from one side of the rectangle. He doesn't stop until there is nothing left! Keep track of how many of each size square Westley made.

Fancy! We've provided you with your very own 200 mm-by-88 mm rectangle on a handout! You can cut or shade the squares—your choice.

2. What is the greatest common divisor of 200 and 88?

3. Here are some calculations that start with 200 and 88. What's going on here?

$$
\begin{aligned}
200 &= 2 \cdot 88 + 24 \\
88 &= 3 \cdot 24 + 16 \\
24 &= 1 \cdot 16 + 8 \\
16 &= 2 \cdot 8 \quad + 0
\end{aligned}
$$

Important Stuff

4. Draw a 16-by-5 rectangle on some grid paper and Westley it up: remove the largest square that you can from one side of your rectangle. Keep removing the largest possible square until there is nothing left.

Careful! We are in the Old West. If you say "draw" too loudly, there might be trouble. This shouldn't be a problem before noon.

- **a.** How many of each size square did you remove?
- **b.** How big was the final square?
- **c.** What is the GCD of 16 and 5?
- **d.** Write calculations like the ones in Problem 3 for 16 and 5.

5. Repeat Problem 4 with a 28-by-12 rectangle.

6. Write each improper fraction as a mixed number in lowest terms.

An improper fraction might set off illegal fireworks that land in other people's hot tubs.

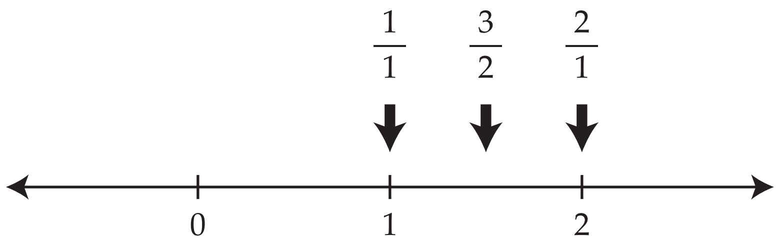
- **a.** $\frac{16}{5}$
- **b.** $\frac{28}{12}$

7. Start with 0 and 1, then keep adding two terms to get the next:

$$0, 1, 1, 2, 3, 5, 8, 13, \ldots$$

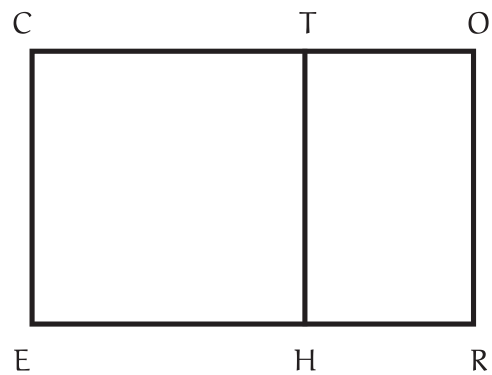
Use consecutive Fibonacci numbers (value/previous value) to make fractions and place these fractions on this number line. Keep placing! What do you notice?

Eventually these numbers start multiplying like rabbits. Wait, no, they're adding like rabbits.



8. You can take any non-square rectangle and chop a square out of it! In the diagram below, THEC is a square chopped out of rectangle CORE.

A *scaled copy* is like Ant-Man. Or there's a Wasp now? Shouldn't there only be half of them now? Anyway.



a. Suppose $EH = 2$ and $HR = 1$. Is rectangle ROTH a scaled copy of the original rectangle CORE?

b. Suppose $EH = 3$ and $HR = 2$. Is ROTH a scaled copy of CORE this time?

c. What if $EH = 5$ and $HR = 3$? Well, shoot.

d. If $EH = x$ and $HR = 1$, write a proportion that would have to be true if ROTH is a scaled copy of CORE. You don't have to solve it.

This one doesn't work either?? EH.

Tough Stuff

21. What's interesting about the fraction $\frac{100}{9899}$? Can you find any other super-interesting fraction friends?

No! No! It's too soon for the finale!

22. **a.** Westley uses his square-cutting method on a 6-by-5 rectangle, ending up with 6 squares. Find a way to partition this rectangle using fewer than 6 squares, and prove that it can't be done with even fewer.

b. Partition the 200-by-88 rectangle using the fewest number of squares possible.

c. Write an algorithm that will partition a m-by-n rectangle into the fewest number of squares.

Okay, it's fine, maybe people won't even notice the finale happened in the middle.

(This will make a lot more sense if you watched the fireworks last night.)

Neat Stuff

9. **a.** Hey, what was that smallest (denominator) fraction within $\frac{1}{100}$ of $\frac{1}{2}$ that wasn't exactly $\frac{1}{2}$?

b. . . . and that smallest fraction within $\frac{1}{100}$ of $\frac{1}{3}$?

c. Do these answers have anything in common?

d. Find the smallest fraction within $\frac{1}{1000}$ of $\frac{1}{13}$.

Okay, phew, back to normal.

10. **a.** What is the GCD of 495 and 132?

b. What is the GCD of $(495 - 132)$ and 132?

c. What is the GCD of $(495 - 3 \cdot 132)$ and 132?

d. What happens if you work through the Opener's square cutting with a 495-by-132 rectangle?

11. Joshua starts with a rectangle that is exactly $\sqrt{3}$ by 1, then starts square cutting. What happens?

Assume that Joshua can cut really, really, really small things accurately.

12. Find the rational number $\frac{p}{q}$ with smallest denominator $q > 0$ such that $\frac{p}{q}$ is between $\frac{17}{92}$ and $\frac{5}{27}$.

13. Find an integer solution to each equation, or explain why a solution cannot exist.

a. $3x + 5y = 1$

b. $3x + 6y = 1$

c. $3x + 7y = 1$

d. $29x + 13y = 1$

e. $154x + 69y = 1$

f. $495x + 132y = 1$

14. Take the diagram from Problem 8 and cut another square from rectangle ROTH. If $EH = x$ and $HR = 1$, determine another proportion that would have be true for the smallest length of the new rectangle formed.

Behold the mighty Thor! . . . no, wait, it's just his brother ROTH. His brother is the God of Retirement Planning.

15. **a.** Prove that $\text{GCD}(a, b) = \text{GCD}(a - b, b)$.

b. Prove that if r is the remainder when you divide a by b, that $\text{GCD}(a, b) = \text{GCD}(d, b)$.

c. What does this mean for Westley's square cutting?

16. Peter has a room that is 22-by-7. And Peter has a room that is 7π-by-7.

Peter and Peter each fire lasers from one corner. Compare the trajectories of the lasers.

You can see more of their adventures on Nickelodeon, Saturdays at 8 pm, 7 Central.

17. Find the GCD of $x^4 - 3x^3 - 6x^2 + 29x - 21$ and $x^5 + 7x^4 + 10x^3 - 18x^2 - 27x + 27$.

18. Find the GCD of $89 + 18i$ and $70 + 9i$.

19. Take the work of Problem 9 and apply it to π and $\frac{22}{7}$: what is the smallest (denominator) fraction within $|\frac{22}{7} - \pi|$ of π? Then try again with π and $\frac{355}{113}$.

20. **a.** Find all the ways to write integers a and b, with $a \leqslant b$, so that

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{4}$$

b. Find a rule in terms of n for the number of ways to write integers a and b, with $a \leqslant b$, so that

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{n}$$

Alright, fire up the Tough Stuff! Problems 21 and 22, go! Wait what?