Day 5: Platypi Ease My Dreadful Aching Stomach

Opener

1. Michelle swears by her home remedy for colds, which consists of 2 tablespoons of ipecac syrup and 3 tablespoons of prune juice. The other Michelle swears by her home remedy for stomach aches, which consists of 5 tablespoons of ipecac syrup and 7 tablespoons of prune juice.

a. Poor Zach has both a cold and a stomach ache, so he makes both recipes and throws it together in a bowl. What is the ratio of ipecac syrup to prune juice in the resulting concoction?

Ah, the painful memories of Day 2 return.

b. Poor Stever has a really bad cold and a stomach ache, so he doubles the cold remedy recipe and throws it in a big bowl with the stomach ache remedy. What is the ratio of ipecac syrup to prune juice in the resulting concoction?

It's possible that this was all caused by Aunt Sally. Please excuse her.

c. Suppose x multiples of the cold remedy recipe are added to y multiples of the stomach ache remedy. Give some examples of what might happen. What is the smallest possible ratio of ipecac syrup to prune juice? The largest?

Important Stuff

2. a. Use mixture concepts to explain why

$$\frac{1}{3} < \frac{3}{8} < \frac{2}{5}$$

b. If $\frac{a}{b} < \frac{c}{d}$ with b and d positive, explain why

$$\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$$

3. Armed with the knowledge of mixing, you can build any fraction! Start with pure prune juice on the left and pure ipecac syrup on the right:

You'll have to pretend the fraction $\frac{1}{0}$ is okay here. Thinking of it as pure prune juice . . . probably makes it worse.

$$\frac{0}{1} \qquad\qquad\qquad\qquad \frac{1}{0}$$

Mixing neighbors produces new fractions. Mixing produces $\frac{1}{1}$:

$$\frac{0}{1} \qquad\qquad \frac{1}{1} \qquad\qquad \frac{1}{0}$$

Mixing neighbors produces two new fractions:

$$\frac{0}{1} \qquad \frac{1}{2} \qquad \frac{1}{1} \qquad \frac{2}{1} \qquad \frac{1}{0}$$

What four new fractions can be added through mixing in the next phase?

No neighbor, no mix. No shoes, no service. No muss, no fuss. No more, no less. No harm, no foul. No pain, no gain. No credit, no problem. No justice, no peace. No woman, no cry. No diggity, no doubt. No tea, no shade. No guts, no glory. No runs, no hits, no errors. No I don't want, no scrubs. Mo money, mo problems.

4. Describe the mixing you will have to do to make $\frac{8}{5}$ and to make $\frac{13}{8}$. Don't bother describing the rest of the full set of numbers involved.

5. **a.** Rewrite this as a single fraction in lowest terms.

$$1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4}}}$$

This type of expression is called a *continued fraction*. To be continued

b. Do some Westley-style square cutting using a rectangle whose width is the numerator of your answer above and whose length is the denominator.

c. How do the numbers from the Euclidean Algorithm connect with the numbers in the continued fraction?

. . . the expression involves multiple nested fraction bars. To learn more about continued fractions, check Wikipedia sometime after the next two weeks!

6. Consider a 121-by-38 rectangle. Keep chopping off the largest square possible until nothing is left.

a. How many squares did you chop off in each phase?

b. Write the continued fraction for $\frac{121}{38}$.

c. What is the GCD of 121 and 38?

7. Let's stop the fraction in Problem 5 after each step, and compare each value to $\frac{43}{30}$:

$$1, 1+\frac{1}{2}, 1+\frac{1}{2+\frac{1}{3}}$$

What's going on?

I say hey.

Neat Stuff

8. On Day 2 you associated the number $\frac{22}{7}$ with the slope of the line passing through the origin and the lattice point $(7, 22)$.

a. Draw a coordinate plane illustrating the fractions $\frac{2}{3}$, $\frac{5}{7}$, 2, and $\frac{4}{3}$ as slopes, in a similar manner.

b. What is the fraction with smallest denominator that is between $\frac{2}{3}$ and $\frac{5}{7}$?

Stever, tell us all about it!

c. How can you use the visual of the coordinate plane to show the order of fractions?

9. **a.** Poor Lorena has both a cold and a stomach ache, so he mixes an *equal amount* of both remedies together. What is the ratio of ipecac syrup to prune juice in the resulting concoction?

Hey, just dropping in to let you know the answer to this question is *not* $\frac{29}{42}$. Yeah, ok bye.

b. Poor Scott has a really bad cold and a stomach ache, so he mixes two parts of the cold remedy with one part of the stomach ache remedy. What is the ratio of ipecac syrup to prune juice in the resulting concoction?

I'm beginning to think these concoctions might not be improving things for these poor folks.

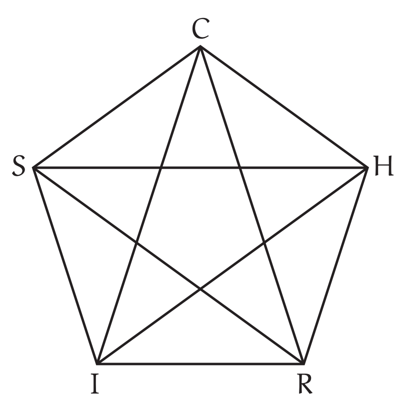
10. Here's a continued fraction:

$$3+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3}}}$$

If any of the threes changed to a four, the value of the continued fraction would change. Which would cause the most change? The least change?

Threes is so much better than 2048.

11. Pentagon CHRIS is regular.



a. Suppose $IR = 1$. What is the length of CI?

b. Look for other places where the ratio CI/IR appears in this figure.

Look for similar triangles instead of using trigonometry. You may want to label some more points, too.

12. Make a regular octagon in the coordinate plane using only lattice points, or explain why it can't be done.

A regular octagon has a high-fiber diet, and all its side lengths and angle measures are the same.

13. Make an equilateral triangle in the coordinate plane using only lattice points, or create a triangle using only lattice points that is a close to equilateral as possible.

14. Go back to the fractions you found on Day 1 as approximations for π. Determine how these fractions would be built using the mixing methods of Problem 3. Notice anything?

Baby go back! These fractions have pretty big bottoms, if you know what I'm saying. 24 over 36? Only if it reduces to two-thirds.

15. Start with any pair of integers from 0 to 9. Add them, taking only the units digit. Then keep going, adding the most recent two terms.

$$4, 8, 2, 0, 2, 2 \ldots$$

a. What happens?
b. How many different "chains" are there, and what is in 'em?

If 2 Chainz were here, he would guess at this problem, and he would be wrong.

16. What is the value of

$$x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$$

17. When Collin plays Linda in the World Cup Final, what will the scoreboard display, and what will the unused letters in the names spell out?

Tough Stuff

18. Pack a 43-by-30 rectangle with as few squares as possible. How many squares is minimal?

19. Consider an infinite set of squares where square n has side length $\frac{1}{n}$. It turns out these squares have a finite total area! That means they can all fit in a 1-by-x rectangle. Find the smallest possible value of x.

If you really want to feel weird, compute the total perimeter of these squares.

20. Find the continued fraction for $\sqrt[3]{2}$.