# Day 2: Slhi'unfg

## Opener

**1.** Can perfect shuffles restore a deck with 9 cards to its original state? If so, how many perfect shuffles does it take? If not, why not?

"Slhiunfg" is the German word for "shuffling", and also once appeared in an HP Lovecraft story.

Split the cards 5-and-4, and keep the top card on top.

### Important Stuff

**2.** Working with your table, fill in a whole lot of *this* table:

`http://www.tinyurl.com/perfectshuffle`

The file is *in* the computer! Oh, only one computer per table, please.

**3.** Find the units digit of each annoying calculation. Put those calculators away!

**a.** $2314 \cdot 426 + 573 \cdot 234$
**b.** $(46 + 1)(46 + 2)(46 + 3)(46 + 4)(46 + 5)$
**c.** $71^4 \cdot 73^4 \cdot 77^4 \cdot 79^4$

The *units digit* of 90210 is 0, matching Brenda's IQ. We'll miss you, Dylan.

**4.** Find all possible values for the units digit of each person's positive integer.

**a.** Andrew: "When you add 5 to my number, it ends in a 2."
**b.** Bethanne: "When you multiply my number by 3, it ends in a 7."
**c.** Carol: "When you multiply my number by 6, it ends in a 4."
**d.** David: "When you multiply my number by 5, it ends in a 3. Yup."

You might not want to write a complete list . . .

**5.** Unlike "base 10", in *mod 10* the only numbers are the remainders when you divide by 10. In mod 10, 6 + 5 = 1 because 1 is the remainder when 6 + 5 is divided by 10.

Answer all these questions in mod 10.

**a.** $2 + 2 = \square$
**b.** $3 \cdot 4 = \square$
**c.** $\square + 5 = 2$
**d.** $4 \cdot \square = 2$
**e.** $5 \cdot \square = 3$
**f.** $\square^4 = 1$

This is sometimes called *modular arithmetic*. Clock arithmetic is mod 12. Four hours from now, it will be close to 1 o'clock. Apparently time is Irish.

That last one says box to the fourth power, by the way.

**6.** Repeat the previous problem, except this time do the arithmetic in *mod 7* instead of mod 10.

Good news: there are only 7 numbers in mod 7. Bad news: in mod 7, every Monday is the same.

. . . sorry, I'm being told there was a one-character typo in the top note.

**7.** Here's a deck of 18 cards in its starting configuration.

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

Perform perfect shuffles and write down where the cards are located after each shuffle.

- **a.** What is the path taken by card 1?
- **b.** . . . by card 2?
- **c.** . . . by card 3?

Poor card 1, always waiting for the deck to be cut or a mistaken shuffle.

**8.** Here's a deck of 18 cards in its starting configuration.

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |

Perform perfect shuffles and write down where the cards are located after each shuffle.

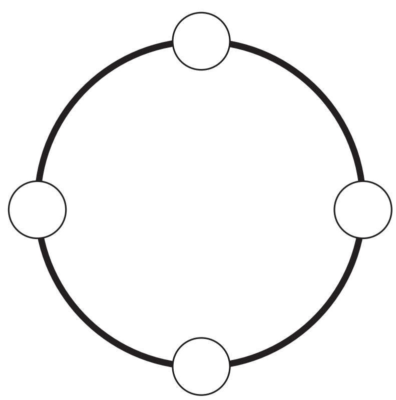
- **a.** What is the path taken by card 0?
- **b.** . . . by card 1?
- **c.** . . . by card 2?

Poor card 17, watching all the younger cards mingling.

## Neat Stuff

**9.** Rob is making a pile of bracelets for the students in his class. Each bracelet has four beads on it. Rob has forest green beads and lime green beads.

The school, Green Academy, loves its football team, the Little Green Men. The band mostly plays Green Day, but also some of Al Green's smooth jazz, some family-friendly Cee Lo, and one song by The Muppets. They care a lot about the environment. Favorite movies involve a book, a mile, a lantern, a beret, some tomatoes, some sort of food product non-beloved by Charlton Heston, and Al Gore. They've only got one favorite math theorem, and one favorite dog treat, and one favorite baseball stadium wall. The clocks at this school all run on GMT.



How many unique bracelets can Rob make? If you can turn and/or flip over one bracelet and make it look like another, then we consider them to be the same (non-unique) bracelet design.

**10.** Go back to the big table that we all filled in together in Problem 2. What cool patterns do you notice?

Some examples of uncool patterns:

"I noticed that most of the values in the table were numbers."

"All the digits in the table could also be found on a computer keyboard."

"Some of the numbers in the table were bigger than others, while others were smaller."

**11.** We overheard Amari and Grace still yelling about whether or not the number .99999... was equal to 1. Is it? Be convincing.

**12.** Write each base-10 fraction as a base-3 "decimal". Some of the answers are already given, in which case—awesome!

**a.** $\frac{1}{13} = 0.\overline{002}_3$

**b.** $\frac{2}{13}$

**c.** $\frac{3}{13}$

**d.** $\frac{9}{13}$

**e.** $\frac{10}{13}$

**f.** $\frac{1}{7} = 0.\overline{010212}_3$

**g.** $\frac{3}{7}$

**h.** $\frac{9}{7}$

**i.** $\frac{6}{7}$

Get strategic on these problems: be as lazy as possible. Use tech only if you really have to here.

**13.** Write the base-10 decimal expansion of

$$\frac{1}{142857}$$

Are there any more interesting ones like this?

**14.** Repeat Problem 9, but now Rob's bracelets have three beads (equally-spaced) each. What about for five beads? Figure out a general rule for $n$ beads.

**15.** If $\frac{1}{n}$ terminates in base 10, explain how you could determine the length of the decimal based on $n$, without doing any long division.

You now know the entire plot of the horrible movie *Terminator 1/4: 0.25 Day*.

**16.** Sierra wonders what kinds of behavior can happen with the base-10 decimal expansion of $\frac{1}{n}$. Be as specific as possible!

**17.** Stephen wonders what kinds of behavior can happen with the base-3 decimal expansion of $\frac{1}{n}$.

**18.** **a.** Suppose $ab = 0$ in mod 10. What does this tell you about $a$ and $b$?

**b.** Suppose $cd = 0$ in mod 7. What does this tell you about $c$ and $d$?

It tells you that $a$ through $d$ hog the spotlight too much. No love for the middle of the alphabet in algebra.

**19.** Investigate shuffling decks of cards into three piles instead of two. What are the options? Does it "work" like it does with two piles?

**20.** **a.** Investigate the base-10 decimal expansions of $\frac{n}{41}$ for different choices of $n$. What happens?

**b.** Investigate the *base-3* expansions of $\frac{n}{41}$ for different choices of $n$. What happens?

The fraction $\frac{n}{41}$ is still in base 10 here, so don't convert 41 to some other number.

**21.** **a.** Find all positive integers $n$ so that the base-10 decimal expansion of $\frac{1}{n}$ repeats in exactly 4 digits.

**b.** Find all positive integers $n$ so that the base-3 "decimal" expansion of $\frac{1}{n}$ repeats in exactly 5 digits.

**22.** Write 223 and 15.125 in base 2. Then write them in base $\sqrt{2}$. How cool is that?!

While this problem is cooler than most math, the Supreme Court recently ruled that math cannot actually be cool.

### Tough Stuff

**23.** Faith has an octahedron, and she wants to color its vertices with two different colors. How many unique colorings are possible? By "unique" we mean that you can't make one look like the other through a re-orientation.

**24.** What about edges? Look for comparisons to Day 1's work on cubes.

Edges? Edges? We don't need no stinkin' edges!