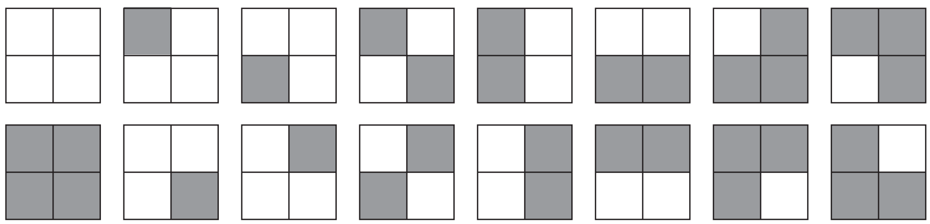
Day 3: Suln'hfig

Opener

Today's title is a rejected name of a Sesame Street character. The character's real name is much harder to spell. Wait, no, it's actually the Klingon word for killing someone while shuffling.

1. Spencer sells customizable 2×2 chessboards, where each square can be white or black. He says he has to make 2^4 different chessboards because there are two colors to choose from, and four squares that can be independently colored. And here they are:



The other Spencer points out that Spencer doesn't have to make all 16 designs above because he can rotate the chessboard by 90°, 180°, or 270°.

Watch them both on the new Court TV drama *Spencer vs. Spencer*! Both of them are for hire, but they still find time to operate a chain of gift shops.

a. What is the smallest number of chessboards that Spencer has to make to be able to get all of the designs above? Or, how many unique colorings of this chessboard exist? Two designs aren't unique if they are rotated versions of each other.

b. Circle the sets of designs above that are considered to be the same via some rotation.

Important Stuff

2. These are the *entire tables* for addition and multiplication in mod 7. FINISH THEM!!!

Today in feeling old: they're now up to Mortal Kombat 11. Fortunately, in mod 7, they're only up to Mortal Kombat 4.

+	0	1	2	3	4	5	6
0							
1							
2				5			
3							
4							
5			0				
6							5

×	0	1	2	3	4	5	6
0							
1							
2				6			
3	0						
4							
5			3				
6							1

. . . sorry, I'm being told there was a one-character typo in the top note.

3. What's the difference between *mod 7* and *base 7*? Write a brief explanation (with a numerical example) that a middle-school student could understand.

Bonus points if you can call a middle school student and tell it to them without them hanging up or discussing Fortnite.

4. Use the tables you built to find all solutions to each equation *in mod 7*. Some equations may have more than one solution, while others may have none.

a. $5 + a = 4$
b. $4 \cdot b = 3$
c. $(5 \cdot c) + 6 = 1$
d. $d^2 - 4 = 0$

5. Complete this table, dividing the work among friends!

And foes, I suppose. Hint: Let the foes do 19 and 25.

n	Powers of 2 in mod n	Cycle Length
7	$1, 2, 4, 1, 2, 4, 1, \ldots$	3
9		
11		
13		
15		
17		
19		
21		
23		
25		

6. Describe the path taken by the top moving card when you perfectly shuffle a 14-card deck.

`http://bit.ly/14cards`

Dang, I thought we were done with shuffling. It seems like we shuffle at least once every 24 hours.

7. Describe the path taken by the top moving card when you perfectly shuffle a 22-card deck.

`http://bit.ly/22cards`

8. Today's handout includes the number of shuffles needed for some different deck sizes. Look for some connections to today's work, and see if you can explain them!

Neat Stuff

9. Are there negative numbers in mod 7? Does any number behave like -1?

Sub-zero says hi.

10. Patricia describes another way to shuffle a deck. After creating two equal piles, her first card comes from the pile in her *right* hand instead of her left. For example:

$$123456 \Rightarrow 415263$$

How does this type of shuffling compare to the usual perfect shuffle? What is the same and what is different?

11. Sometimes while shuffling, the deck completely flips. When this happens, all cards appear in reverse order, except for the very top and bottom cards.

a. Find two examples of deck sizes when this occurs.
b. Luke claims that when this happens, that was a halfway point to the shuffling. Is this true? Explain why or why not.
c. Troy claims that this happens at the halfway point of all perfect shuffle sequences. Is this true? Explain why or why not.

Flipping your deck while shuffling sounds like one of the greatest breakdancing moves ever. Breakdancing hasn't been the same since they canceled production on Breakin' 3: The Boogaloo Kid.

12. **a.** Kennya is calculating the powers of 3 in mod 100. Compute the next three entries in the sequence.

$$1, 3, 9, 27, 81, 43, 29 \dots$$

b. Compute the sequence of powers of 3 in mod 3. Uhh.
c. What about mod 7? What about other mods?

13. **a.** Write $\frac{1}{7}$ as a base-3 "decimal".
b. Write $\frac{1}{100}$ as a base-3 "decimal".

This is not $\frac{1}{9}$. If you'd rather, you can think of it as $\frac{1}{10201}$. Hey, 100 is a palindrome in base 3, nifty.

14. Alissa, Benjamin, and Caleb were standing in the lunch line and had an idea. If any two neighbors switch places, it would create a different arrangement... like

$$ABC \Rightarrow ACB$$

They decided to make a big graph of all six ways they could be arranged, and all the connections that could lead from one way to another. Your turn!

Note that only *neighbors* may switch places. Alissa and Caleb can trade places, but not as the first move.

15. Danny gets in the back of the line behind Alissa, Benjamin, and Caleb, and they realize they're going to be stuck making a much larger graph. Good luck!

This graph will contain lots of little copies of the last graph! Neat.

16. If a number can be represented as a repeating decimal in base 10, does it have to be a repeating decimal in every other base? If yes, explain why. If no, are there any particular bases in which it *must* be a repeating decimal?

17. Predict the length of the base-10 repeating decimal expansion of $\frac{1}{107}$, then see if you were right.

Some of these problems are as confusing as the fact that Steph Curry now hosts a mini-golf game show.

Tough Stuff

18. Predict the length of the *base 2* repeating decimal expansion of $\frac{1}{107}$, then see if you were right.

19. What powers of 10 are palindromes in base 3?

20. For even n, the maximum number of perfect shuffles needed to restore a deck with n cards to its original state appears to be $n-2$. Find a rule that tells you when an n-card deck will require $n-2$ perfect shuffles.

21. *It's p-adic number time!* Every 2-adic positive integer looks like it normally does in base 2, except it has an infinite string of zeros to the left. For example

$$7 = ...000000000000000111.$$

Where he at, where he at . . . p-adic numbers, p-adic numbers, p-adic numbers with a baseball bat!

a. Verify that $7+4=11$ using 2-adic arithmetic.
b. What about subtraction? Try $4-3$, and then try $3-4$.
c. Compute the sum

Oh *man* are you going to have to do a lot of borrowing!

$$...111111111111111. + ...0000000000001.$$

What number is represented by ...11111111111111.?
d. Compute this sum using 2-adic arithmetic:

$$1+2+4+8+16+\cdots=?$$

22. **a.** Find all solutions to $x^2-6x+8=0$ in mod 105 without use of any technology. There's a lot of them.
b. Find all solutions to $x^2-6x+8=0$ in mod 1155 without use of any technology.

Tell you what, we'll give you two solutions: $x=2$ and $x=4$. See, now it's not nearly as tough.