Day 4: Shuf'ling Shuf'ling

Opener

1. Each person at your table should pick one of these cards: 3 of ♠ (spades), 9 of ♠, King of ♠, 5 of ♡, King of ♣ (clubs), Queen of ♢. Use this web page to track the location of their card within a standard deck of cards as they undergo perfect shuffles.

`http://tinyurl.com/pcmi52cards`

Then, compare notes with each other.

If there are fewer than six people in your group, each person can track more than one card.

Hm. Are we really going to be shuffling *every day*? At least we should have appropriate music.

Important Stuff

2. Renu sells a line of customizable products featuring equilateral triangles in which each side of the triangle can be red, blue, or green.

Renu allows customers to repeat colors or not. It's customizable!

- **a.** Renu sells customizable t-shirts featuring these equilateral triangles. How many unique t-shirt designs does she need to keep in stock?
- **b.** Renu sells customizable equilateral triangle stickers. How many unique designs will she need to keep in stock, given that the stickers can be rotated? (Two sticker designs are considered non-unique if they are rotated versions of each other.)
- **c.** Renu sells customizable equilateral triangle metal paperweights. How many unique designs will she need to keep in stock, given that the paperweights can be rotated and flipped over?

3. Again, use `http://tinyurl.com/pcmi52cards` to...

- **a.** ... track the numerical positions of the 2 of ♠.
- **b.** ... track the numerical positions of the 4 of ♠.
- **c.** ... track the numerical positions of the 6 of ♠.

Do not use this website to get up, get down, or to put your hands up to the sound.

4.
- **a.** What are the powers of 2 in mod 51?
- **b.** Multiply the numbers in part a by 3 in mod 51.
- **c.** Multiply the numbers in part a by 5 in mod 51.
- **d.** What do you notice?
- **e.** Multiply those numbers by 17 in mod 51. Huh.

5. Explain why, in a 52-card deck, the top moving card returns to its original position after 8 perfect shuffles.

It's times like you would think a Shufflebot would be useful. Sadly, Shufflebot is only programmed to dance and to apologize.

Neat Stuff

6. Use `http://go.edc.org/21cards` to keep track of the position of each card in a 21-card deck as it undergoes perfect shuffles. Complete this table.

Card No.	Positions	Cycle Length	Card No.	Positions	Cycle Length
0	$0, 0, 0 \ldots$	1	11		
1			12		
2			13		
3			14		
4	$4, 8, 16, 11, 1, 2, 4, \ldots$	6	15		
5			16	$16, 11, 1, 2, 4, 8, 16, \ldots$	6
6			17		
7			18		
8			19		
9			20		
10					

7. All 52 cards from the deck we saw on Monday have a cycle. List all 52 cycles. Maybe there is some way to do it without listing the entire cycle for every single card?

8. **a.** In Problem 6, card numbers 4 and 16 were essentially on the same cycle. How many unique cycles does the 21-card deck have?
b. How many unique cycles does a standard 52-card deck have?

LMFAO definitely had some unique cycles, especially when they ran that failed kids' toy store with Schwarz. Leopard prints and shufflebots? No thank you.

9. There are 21 cycle lengths in Problem 6. Add up the *reciprocals* of all 21 numbers . . . surely, this will be a mess . . . oh. What up with that? Look for an explanation.

And we gonna make you lose your mind . . . we just wanna see ya . . . flip that. Doo doo doo do DOO da doo . . .

10. So $2^8 = 1$ in mod 51. All this actually proves is that the *first* moving card returns to its original position after 8 shuffles. Complete the proof by showing that card n, regardless of n, also returns to its original position after 8 shuffles.

11. Pick some deck sizes that you shuffled. Use this to predict the number of powers of 2 in mod n for various n, then verify your prediction by calculating.

Party math is in the house tonight . . . everybody just have a good time.

12. Our favorite repeater, $\frac{1}{7}$, can be written as a "decimal" in each base between base 2 and base 10. Find each expansion and see if they have anything in common.

Step up fast, and be the first one at your table to do base 7.

13. Investigate any connection between the number of powers of 2 in prime mods and the number of powers of 2 in composite mods. Look for an explanation or proof of what you find.

14. Peter, Quiana, and Rasa are waiting in line, wondering if they can get to any arrangement through these two rules:

- The person in the back of the group may jump to the front: $PQR \Rightarrow RPQ$
- The two people at the front of the group may swap places: $PQR \Rightarrow QPR$

Swaps, swaps, swaps! Swaps swaps swaps!

Can all six possible arrangements be made? Make a graph illustrating the options.

15. Shanel joins the back of the group. Under the same rules, decide whether or not all 24 possible arrangements can be made, and make a graph illustrating the options.

One such arrangement is a tribute to the Roman army, the morning after the invasion of the Roman candles!

16. A perfect 52-card shuffle doesn't have a flipped deck at 4 shuffles; we would have noticed that. But does anything interesting happen after the 4th shuffle? Look carefully and compare the deck after 4 shuffles to the original deck. What's the dilly-o?

The observation here may be easier with two decks of cards; one to shuffle, and one to leave in the original setup.

17. Justin handed us a note that said:

$$10^2 + 11^2 + 110^2 = 111^2.$$

a. Surely the numbers 10, 11, 110, and 111 in the note are in base 2. Check to see if the statement is true in base 2.

b. Hey wait, maybe those numbers are in base 3. Check to see if the statement is true in base 3.

c. Oh, hm, maybe it was in base 4.
d. Sorry, it was actually in base n. What!

All your bases are belong to us.

18. *It's p-adic number time!* In 3-adic numbers, non-negative integers are written in base 3 with leading zeros:

$$16 = ...00000000000121.$$

a. Try $16 - 9$. Hey, that wasn't so bad!
b. Try $16 - 17$. Oh dear.
c. In 3-adic numbers, what real number has the same value of $1 + 3 + 9 + 27 + 81 + \cdots$?

Aww yeah! The p-adic numbers make as much sense as most LMFAO videos.

This problem's got that devilish flow rock and roll no halo.

Tough Stuff

19. A stack of 27 little cubes is built to make a bigger 3x3x3 cube. Then, two of the little cubes are removed. Ignoring any gravity effect, how many unique shapes are possible?

It works according to the conversion 27 cubes = 1 Rubik.

20. The length of the repeating decimal for $\frac{1}{2}$ in base p, where p is prime, is sometimes even and sometimes odd. When? Find a rule and perhaps a proof even?

The first person to find and prove this will receive a champagne shower! Offer expires 7/4/2019.

21. For what primes p is there an even length of the repeating decimal for $\frac{1}{5}$ in base p?

22. For what primes p is there an even length of the repeating decimal for $\frac{1}{10}$ in base p?

23. Julia's favorite number is the golden ratio $\phi = \frac{1+\sqrt{5}}{2}$. Julia loves to do arithmetic in base ϕ. The only problem is that even if the only allowable digits in base ϕ are 0 and 1, not every number has a unique representation. Prove that every number has a unique base-ϕ representation if two consecutive 1s are disallowed.

24. Determine and prove the Pythagorean Theorem for p-adic numbers, or decide that this problem is completely bogus and there is no such thing.

It's mathy and you know it.