Day 9: Shuffles, Assorted

Opener

1. Use this set of equations to find the base 2 "decimal" for $\frac{5}{51}$.

$$\begin{aligned} 5 &= 0 \cdot 51 + 5 \\ &\vdots \\ 10 &= 0 \cdot 51 + 10 \\ 20 &= 0 \cdot 51 + 20 \\ 40 &= 0 \cdot 51 + 40 \\ 80 &= 1 \cdot 51 + 29 \\ 58 &= 1 \cdot 51 + \\ &= \quad \cdot 51 + \\ &= \quad \cdot 51 + \\ &= \quad \cdot 51 + 5 \end{aligned}$$

Thank goodness there are no obnoxious giant tables on this problem set. Let's celebrate! As is customary, we gotta get down.

What would William Shatner yell if he was upset with the pedagogical style of a tutoring website? Answer later.

Important Stuff

2. **a.** When you multiply a number by 10 in base 10, what happens to the decimal point?
b. When you multiply a number by 2 in base 2, what happens to the "decimal point"?
c. The base 2 "decimal" for 16/51 is $0.\overline{01010000}_2$. What's the base 2 "decimal" for 32/51?
d. Based on your previous answer, what's the base 2 "decimal" for 13/51?

The *Curly Shuffle* is a 7-cycle on the floor: "Wooooooooo woo woo woo, woo woo woo. Wooooooooo woo woo woo, woo woo woo." See also: Angus Young in Let There Be Rock; Homer Simpson in the "Lisa needs braces" episode.

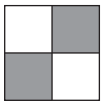
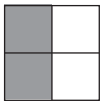
3. **a.** Grahame demands you write all the powers of 2 in mod 51, starting with 1.
b. Take all the powers of 2 in mod 51 and multiply them by 2. What happens?
c. Take all the powers of 2 in mod 51 and multiply them by 5. What happens?
d. Take all the powers of 2 in mod 51 and multiply them by 17. What happens?

Dave and Brian McKnight both agree that the list of powers ends back at 1.

Like Anthrax, you're caught in a mod! Once you're in a pattern you are stuck. Stuck in a coicle, nyuk nyuk nyuk.

4. **a.** Jaime and Jennifer ordered t-shirts with these two logos on the front. Fill in this table for what their t-shirt would look like if their logo was transformed as described.

Hmm, isn't this an obnoxious giant table? **No!** It is fine, just really annoying how it's table-spreading and taking up the whole page. Awww man!

Original	Jaime: 	Jennifer: 
Rot 0° ↺		
Rot 90° ↺		
Rot 180° ↺		
Rot 270° ↺		
Refl 		
Refl 		
Refl 		
Refl 		

Remember that the rotate by $0°$ transformation is an *identity transformation* that doesn't do anything to the object.

Ali: "Now hold it. The *Ali Shuffle* is a dance that will make you scuffle. During the time that I'm doing this shuffle, for a minute, you're going to be confused. You must get in a boxing position, and have a little dance." Howard Cosell – do the voice: "What we've just seen perhaps is the heavyweight champion of the world in what should be his true profession, that of a professional dancer."

b. A *symmetry* of an object is just a transformation of the object that leaves it looking identical. Of all the possible transformations of the 2×2 chessboard, which symmetries does Jaime's design have? Jennifer's?

c. How many total possible symmetries are there? How many does Jaime's design have? How many does Jennifer's design have?

The *Ickey Shuffle* is the most famous touchdown dance of all time. One of the two TV announcers from the movie *Cars* did the Ickey Shuffle after winning a NASCAR race. For more information, watch the "Return of the Shirt" episode from *How I Met Your Mother*. It's . . . wait for it . . .

5. **a.** Without converting to a fraction, find a base-10 decimal such that

$$0.\overline{291594} + 0.\overline{mariah} = 1$$

It's like Mariah's "Get Your Number".

b. Without converting to a fraction, find a base 2 decimal such that

$$0.\overline{011001} + 0.\overline{ishrat} = 1$$

c. In the obnoxious table, how do the base 2 expansions of $\frac{1}{51}$ and $\frac{50}{51}$ compare?

Wait, there's an obnoxious table? Oh no. *NOOOOOO!* Ask Bowen and Darryl for it.

6. Complete the obnoxious table. Be lazy and avoid the use of techmology.

Oh, bother.

7. Describe any patterns you notice in the obnoxious table. What fractions form "cycles", using the same numbers in the same order? Write out each cycle in order, and find all the cycles. How many unique cycles are there?

8. Here is our happy little 52-card deck doing happy little perfect shuffles, just like the very first time we saw it.

`http://tinyurl.com/pcmi52cards`

Follow some cards. Follow some remainders. Figure out how you can use the cards to find the *entire base 2 expansion* for a fraction in the form $\frac{n}{51}$.

Bob Ross's hair was permed so that he could save money on haircuts. Before his art career, he was a sergeant at an Air Force base in Alaska, where presumably he saw many happy mountains and trees. After leaving the military he vowed never to scream again. Bob Ross went on to become a painter beloved by many, including the creators of "Friends", who named David SchwimmerâĂŹs character after him.

Neat Stuff

9. Let's look at some eight-digit numbers in base 2.

a. What number is 00011001_2 in base 10?

b. One of the fractions in the obnoxious table has a base 2 expansion of $0.\overline{00011001}$. Which one? Try to figure out how you might find that fraction without looking at the obnoxious table, and without staring at cards like crazy.

c. Start over with 00101101_2 and $0.\overline{00101101}$.

d. Try it again with 11110000_2 and $0.\overline{11110000}$.

Cupid loves you! To the right . . .

e. What are the largest and smallest possible values of an eight-digit number in base 2?

f. Can you explain why all the entries in the obnoxious table turn out to be multiples of [REDACTED]?

At least it's not an obnoxious multiple.

10. **a.** The base 2 number $110010ab_2$ is a multiple of 5. What are the missing digits, and what multiple of 5 is it?
b. The base 2 number $001101ab_2$ is a multiple of 5. What are the missing digits, and what multiple of 5 is it?
c. Make your own: pick the first six digits of a base 2 number and try to find the missing digits.
d. How does our magic trick work?

. . . legendary!!

11. Based on the card animations, what do you get for the base 2 expansion for $\frac{51}{51}$? Interesting.

What?? You mean it's not magic? No, and neither was getting Lauren to pick the card that does the Cupid Shuffle. Thank you, Lauren!

12. **a.** Take all the powers of 2 in mod 51 and multiply them by k. That was fun. Whee! It will be helpful, we promise.
b. A 52-card deck returns to its original position in 8 perfect shuffles. But some cards return sooner. Find an equation that would be true for any card k that returns to its original position after 2 shuffles, then solve it.
c. Find an equation that would be true for any card k that returns to its original position after 3 shuffles, then solve it.
d. FOUR!

Stay. You only hear what you want to.

Zero bottles of beer on the wall, zero bottles of beer. You take one down and pass it around, 50 more bottles of beer in mod 51.

That's not even a question. But don't worry, be happy.

13. Use ideas from Problem 12 to show that if u is a unit, then u must be part of a cycle that has the same length as the cycle containing 1.

14. A 36-card deck returns to its original position in 12 perfect shuffles. Determine all the cards that return sooner, and the cycle length of each.

15. Here are some shuffle animations. Triple shuffles!!! We promise they're cool.

`http://go.edc.org/3piles`

Go figure stuff out. Decimals in base 3, powers of 3 (in what mod?), cycles, magic tricks, all that jazz. What stays the same? What changes?

From *Hollywood Shuffle*: Ain't nothin' to it, but to do it! This movie featured Keenen Ivory Wayans as "Jheri Curl" (he also wrote the movie).

16. Determine all cycles of cards in the triple shuffle, and the cycle length of each. Given the number of units in the mod, what cycle lengths are possible?

It continues to be the case that you are shuffling in a typical siderial period.

17. **a.** Find a number x so that $x = 1$ in mod 11 and $x = 0$ in mod 13.
b. Find a number y so that $y = 0$ in mod 11 and $y = 1$ in mod 13.
c. Find a number z so that $z = 5$ in mod 11 and $z = 6$ in mod 13.

KHAAAAAANNNNNNNNN!

18. Find a number M so that $M = 2$ in mod 3, $M = 3$ in mod 5, $M = 4$ in mod 7, and $M = 5$ in mod 11. Try this without techmology and with as little guesswork as possible!

Will computahs ever be able to work out what 999999999999999999 multiplied by 9999999 ... yo I haven't finished you don't know what I was going to say ... 99899999999.889988999 9999999999 is?

19. Look back at the table from Day 8 you made for the fractions from $\frac{1}{21}$ through $\frac{20}{21}$. All the digits from 0 through 9 are used, and there are 120 total digits in the repeating parts of the expansions.
a. Which digit is used most frequently in the decimal part of the expansions? Which digit is used least frequently?
b. Does this happen in other mods? Other bases?? So cool.

Okay, it's still not cool, but cool by math standards. Not cool like Kool Moe Dee cool, but hey.

20. Today is 7/12/19, and $7 + 12 = 19$. Oh snap!
a. How many more times this century will there be a day like this? By *this* we mean the next one is August 11, 2019.
b. How many times will Buck Rogers in the 25th Century see a day like this? You may assume that Buck Rogers arrives on January 1, 2401 and remains alive through the entire century.
c. How can the second answer help you check the first?

And the last one is . . . later than the others?

Wow, that is a really cool answer!

Tough Stuff

21. Figure out a way to shuffle cards so that you could use the cards' positions after each shuffle to read off

base-10 decimal expansions of fractions . . . or prove it is impossible to do so.

22. Prove that there are infinitely many primes p for which the decimal representation of $\frac{1}{p}$ has cycle length $p - 1$.

23. Take a grid of circles, 12-by-21, and color each circle with one of four colors. Either find a coloring that does not produce a monochromatic rectangle – a rectangle with all four corners the same color – or prove that such a coloring is impossible.

24. Find a fraction that has the Fibonacci numbers in its decimal expansion:

$$0.00100100200300500801302 1034\ldots$$

25. Find a fraction has the square numbers in its decimal expansion:

$$0.00100400901602503604906 4081\ldots$$

Dang, there's an entire half-page here for another obnoxious table! Womp womp.