Problem Set 1: Happy Pi Day, Chicago Deep Dish Style

Welcome to PCMI! We know you'll learn a great deal of mathematics here—maybe some new tricks, maybe some new perspectives on things with which you're already familiar. A few things you should know about how the class is organized:

- **Don't worry about answering all the questions.** If you're answering every question, that's our fault, not yours.
- **Don't worry about getting to a certain problem number.** Some participants have been known to spend the entire session working on one problem (and perhaps a few of its extensions or consequences).
- **Stop and smell the roses.** Getting the correct answer to a question is not a be-all and end-all in this course. How does the question relate to others you've encountered? How do others think about this question?
- **Be excellent to each other.** Believe that you have something to learn from everyone else. Remember that everyone works at a different pace. Give everyone equal opportunity to express themselves.
- **Teach only if you have to.** You may feel the temptation to teach others in your group. Fight it! We don't mean you should ignore your classmates but give everyone the chance to discover. If you think it's a good time to teach your colleagues about the Law of Cosines, think again: the problems should lead to the appropriate mathematics rather than requiring it.

PCMI teachers have solved two previously unsolved problems presented in these courses.

Opener

Let's watch . . . *Survivor!*

Math teachers ready . . . *go!*

Rules of the game. Figure out what you can! Think about visualizations that make the game and the strategy simpler to understand.

Hopefully you have enough po-po-po-poker chips. If you need more, grab 'em!

1. 21, all same color. Pick 1, 2, or 3. Last pick wins.

2. 21, all same color. Pick 1, 2, or 3. Last pick *loses*.

3. 21, all same color. Pick 2 or 3, unless there's 1 left. Last pick wins.

4. 21, all same color. Pick as many as you want of the same color. Last pick wins.

5. 13 of one color, 8 of another. Pick one chip of either color. Last pick wins.

6. 13 of one color, 8 of another. Pick one chip of either color *or* one of each color. Last pick wins.

7. 13 of one color, 8 of another. Pick as many as you want of the same color. Last pick wins.

8. 13 of one color, 8 of another. Pick as many as you want of the same color *or* the same number of each color. Last pick wins.

For example, you could leave 10 and 5 after your first pick, but not 10 and 6.

9. 9 of one color, 7 of another, 5 of a third color. Pick as many as you want of the same color. Last pick wins.

10. 9 of one color, 7 of another, 5 of a third color. Pick as many as you want of the same color *or* the same number of . . . hm, there's multiple options now. Last pick wins.

11. For the game in problem 8, (1,2) is a "losing position" because the other player can definitely beat you. Make a list of losing positions (a, b) with $a < b$ and figure out what you can.

A longer list makes it more likely you'll find some amazing stuff.