Problem Set 2: Triple Trouble

Opener

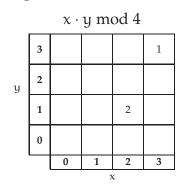
A *Pythagorean triple* is three integers (a, b, c) which are side lengths of a right triangle, with hypotenuse length c.

Find a boatload of Pythagorean triples, and classify them into categories.

If you're interested, you can add your triples to this list:

go.edc.org/chicago-triples

- It's easy as (a, b, c). It's easy as (1, 2, 3).
- *No.* Sorry, Michael, (1,2,3) is not a Pythagorean triple.
- **1.** The value of an integer, mod 4, is its remainder when you divide by 4.
 - **a.** What is $3 + 3 \mod 4$?
 - **b.** What is 3 · 3 mod 4?
 - c. Build addition and multiplication tables for mod 4.



These tables are oriented the way addition and multiplication tables should be oriented! Yeah!

- **2.** A *primitive Pythagorean triple* is a Pythagorean triple with no common factors greater than 1.
 - **a.** Show that every primitive Pythagorean triple has at least one odd number.
 - **b.** Show that in a Pythagorean triple, it isn't possible to have a and b both be odd while c is even.
 - **c.** Show that every primitive Pythagorean triple has exactly two odd numbers.
- **3.** Show that every primitive Pythagorean triple must contain a multiple of .

Primitive Pythagorean triples have discovered the right angle, but not fire.

Oops, forgot the number. Sorry!

- **4.** Find some ways to classify primitive Pythagorean triples, and some ways to generate more of them.
- **5. a.** Can there be two Pythagorean right triangles with the same perimeter?
 - **b.** Fine. Can there be two *primitive* Pythagorean right triangles with the same perimeter?
- **6.** Find *four* different Pythagorean triples with hypotenuse 65, or however many there are. Who knows, there might not even be four.

Whoever wrote this is really lazy. Whoever wrote *this* is also really lazy.

- 7. Start over, and work the same problems finding *Eisenstein* triples. Eisenstein triples have a 60-degree angle opposite *c*, but otherwise everything is the same.
- **8.** Work the same problems finding *anti-Eisenstein* triples. Anti-Eisenstein triples have a 120-degree angle opposite c.
- **9.** A *Matsuura triangle* is a triangle for which there is a point inside the triangle forming 120-degree angles with each vertex, and for which all six segments built from this diagram have integer length. Find the smallest-perimeter Matsuura triangle, or prove no such triangle exists.

Careful, don't put an Eisenstein triple next to an anti-Eisenstein triple! If you do, a plate of antipasto will explode.