Problem Set 2: Triple Trouble

Opener

A *Pythagorean triple* is three integers (a, b, c) which are side lengths of a right triangle, with hypotenuse length c.

It's easy as (a, b, c). It's easy as $(1, 2, 3)$.

No. Sorry, Michael, $(1, 2, 3)$ is not a Pythagorean triple.

Find a boatload of Pythagorean triples, and classify them into categories.

If you're interested, you can add your triples to this list:
go.edc.org/chicago-triples

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1. The value of an integer, mod 4, is its remainder when you divide by 4.

a. What is $3 + 3 \bmod 4$?
b. What is $3 \cdot 3 \bmod 4$?
c. Build addition and multiplication tables for mod 4.

These tables are oriented the way addition and multiplication tables should be oriented! Yeah!

$x + y \mod 4$

y				
3				2
2				
1			3	
0				
x	0	1	2	3

$x \cdot y \mod 4$

y				
3				1
2				
1			2	
0				
x	0	1	2	3

2. A *primitive Pythagorean triple* is a Pythagorean triple with no common factors greater than 1.

Primitive Pythagorean triples have discovered the right angle, but not fire.

a. Show that every primitive Pythagorean triple has at least one odd number.
b. Show that in a Pythagorean triple, it isn't possible to have a and b both be odd while c is even.
c. Show that every primitive Pythagorean triple has exactly two odd numbers.

3. Show that every primitive Pythagorean triple must contain a multiple of   .

Oops, forgot the number. Sorry!

4. Find some ways to classify primitive Pythagorean triples, and some ways to generate more of them.

5. **a.** Can there be two Pythagorean right triangles with the same perimeter?
b. Fine. Can there be two *primitive* Pythagorean right triangles with the same perimeter?

6. Find *four* different Pythagorean triples with hypotenuse 65, or however many there are. Who knows, there might not even be four.

Whoever wrote this is really lazy. Whoever wrote *this* is also really lazy.

7. Start over, and work the same problems finding *Eisenstein* triples. Eisenstein triples have a 60-degree angle opposite c, but otherwise everything is the same.

8. Work the same problems finding *anti-Eisenstein* triples. Anti-Eisenstein triples have a 120-degree angle opposite c.

Careful, don't put an Eisenstein triple next to an anti-Eisenstein triple! If you do, a plate of antipasto will explode.

9. A *Matsuura triangle* is a triangle for which there is a point inside the triangle forming 120-degree angles with each vertex, and for which all six segments built from this diagram have integer length. Find the smallest-perimeter Matsuura triangle, or prove no such triangle exists.