

PCMI December Outreach Weekend

24-25 March 2018

Math!



Survivor Time

URL: go.edc.org/survivor21

Shared Websites:

Projects.ias.edu/PCMI

[Past PCMI problem sets](#)

[Illustrative Mathematics](#)

Past TLP Summer Sessions:

<http://projects.ias.edu/pcmi/hstp/previous.html>



Games Games Games! *What do you notice?*

Wonder? What are your questions?

- Higher numbers, you have to completely work out all the possibilities to see if it breaks down to an earlier smaller case (e.g. Move X is equivalent to Y in the earlier game).
- A lot of our methods involved matching or reacting to the other person to make groups of 4 or 8.
- I wonder if there is some sort of probability equation that would help me solve this? Looking at diagonals (drawing 1 color and then 2 of one color then 3 of one color, etc. set up in a diagonal to eliminate the choices and get down to last 3 – 2 of one color and 1 of another) - referencing problem 9.



Games Games Games! *What do you notice?* *Wonder? What are your questions?*

- We noticed that in #7 you want to leave the other person with an even number when the colors switch from 2 to 1. What is the critical move to make when the transition occurs from 2 colors to 1 color.
- Could this be similar to a permutation equation?



Participant Work

① Pick 1st → only pick 7
Pick 2nd → stick them w/ mult of 4

② Pick 1st → uphill battle, pick 21
7, 2, 3 get to multiple of 4+1
Pick 2nd → pick any, stick w/ multiple of 4+1

③ Label 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21
5x, 5x-1
5x, 5x-1 4, 5, 9, 10, 14, 15, 19
first pick needs to pick 2

④ 4, 8 → first pick 5

⑤ first pick → 21

⑥ first pick automatically wins

⑦ Presented with

1	W
2	same-L dif-W
3	same-W 2-1-W
4	same-L 3-1-W 2-2-L
5	same-W 4-1-L 3-2-W
6	

Survivor

Take Last Flag to win

Team 1 Team 2

2	2	21
1	2	18
2	1	17
1	1	16
2	2	15
1	1	14
2	2	13
1	1	12
2	2	11
1	1	10
2	2	9
1	1	8
2	2	7
1	1	6
2	2	5
1	1	4

Do we assume = knowledge on both teams?

Give 4, 8, 12, 16, 20

• If you go first leave 4 for opponent

• If you go second leave 4+1

④

1 W	6 W	11 W	16 L	21 W
2 W	7 W	12 L	17 W	
3 W	8 L	13 W	18 W	
4 L	9 W	14 W	19 W	
5 W	10 W	15 W	20 L	

no multiples of 4

⑤ take them all

⑥ take one at a time first person will always win

⑦

1 W	6	11	16	21
2	7	12	17	
3	8	13	18	both num are even
4	9	14	19	then you lose
5	10	15	20	

2 one color 1 of the other win
2 of each loss
2 of one & 3 of other win
3 of each win
2 of one & 4 of other

Math 2

1 W 5 W 9 W
2 W 6 W 10 W
3 W 7 W 11 W
4 L 8 L 12 L

Multiplying 4

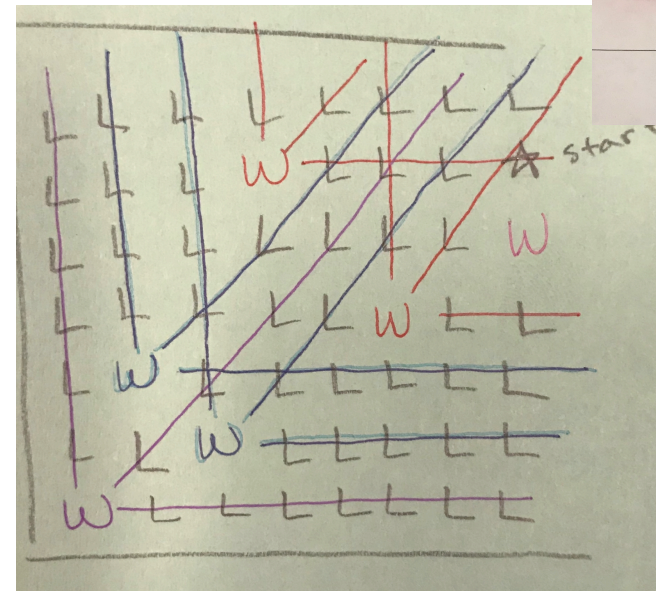
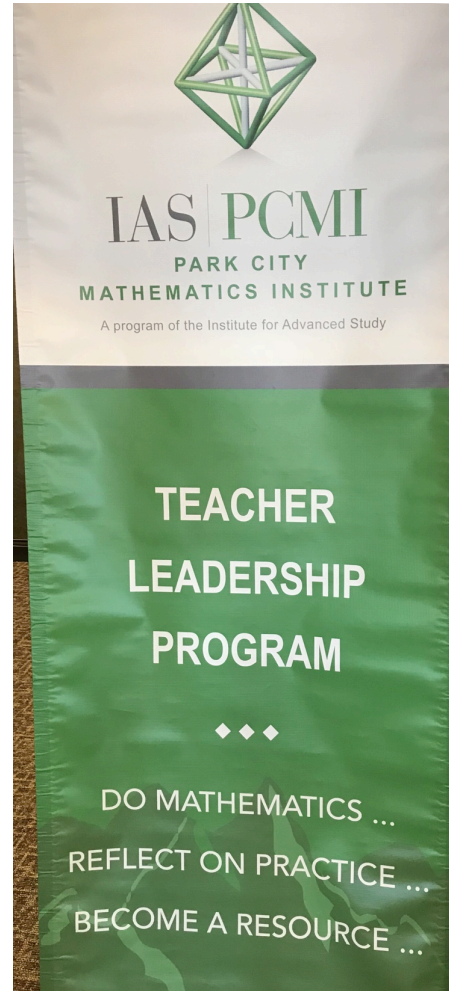
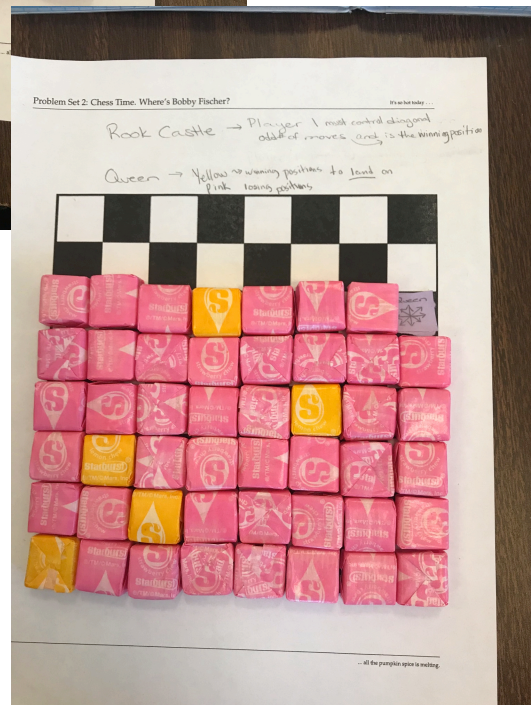
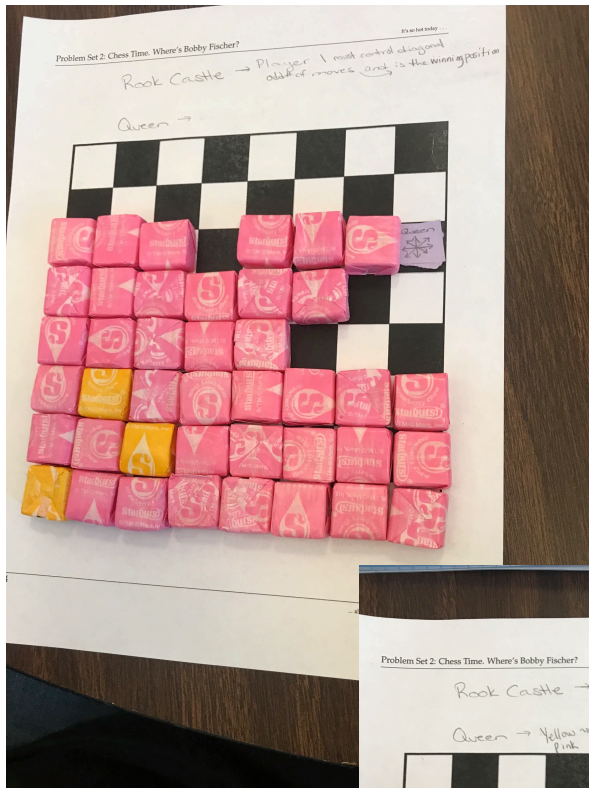
5 W 7
6 W

①
②
2, 1

③
④
⑤
⑥
⑦
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⑪
⑫
⑬
⑭
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Participant Work



2) Chess Time. Math Time

* Be first to an edge, leaving an even # of spaces

Assumptions: follow moving rules for "shaded path" problems

#2	Conjecture	Position (move first)
Rock	Even # of moves - lose	
	Odd # of moves - win	

Player must get to the shaded to guarantee the win

Player who controls the wins.

L	W	L	X
L		L	
L L L L	W		
L W L L			
L L W L			
W L L L L L L L			

Problem Set 2: Chess Time. Where's Bobby Fischer?

even-same L

W
R
L
W
W

2/2-L		
3/2-W	3/3-W	
4/2-W	4/3-	
5/2-W		
6/2-	5/3-	4/4-
7/2-	6/3-	5/4-

5-5-

PS1
#7

PS2
#1

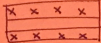
equal


-1 -1


(1,5)	(1,7)
(3,5)	(3,7)
(5,5)	(5,7)
(7,5)	(7,7)
(9,5)	(9,9)
(1,5)	(1,4)
(13,5)	




Chess

①  Starter wins
Get to a win spot 3 or 4

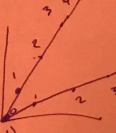
②  starter wins
Get to a diagonal 3 or 4

③  (1,1) (3,2) (6,4) starter wins
(2,3) (4,6) (get to) 3 only (5,8) (8,5)

0 1 2 3

⑤ King Level 1  want to | starter
Level 2 All spots be placed on | wins
even levels

Rook



(0,1)
(3,2)
(6,4)
(8,5)
(11,7)

Chess Tic Fischer?

1. A chess king sits three squares away from the top-left corner of a chessboard. The player who puts the king in the corner first wins. Oh, you'll also need to figure out what means.
2. A chess rook (castle) sits three squares away from the top-right corner of a chessboard. Same rules. Who wins?
3. A chess queen sits three squares below the top-left corner. Same rules. Who wins, and how?
4. Bring your new knowledge to bear on some of the problems you did yesterday. What do you learn?
5. Repeat the king and rook problem, but this time the queen pieces are in 3-D, and they can move in any direction to the north, and south, and east, and west, and how a queen can move.

0 1 2 3

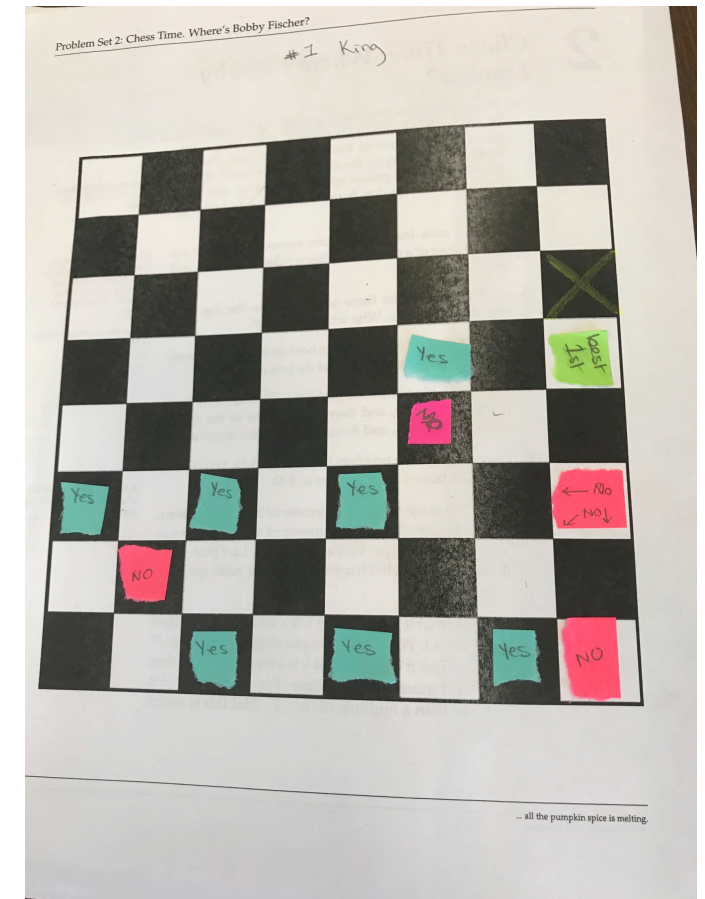
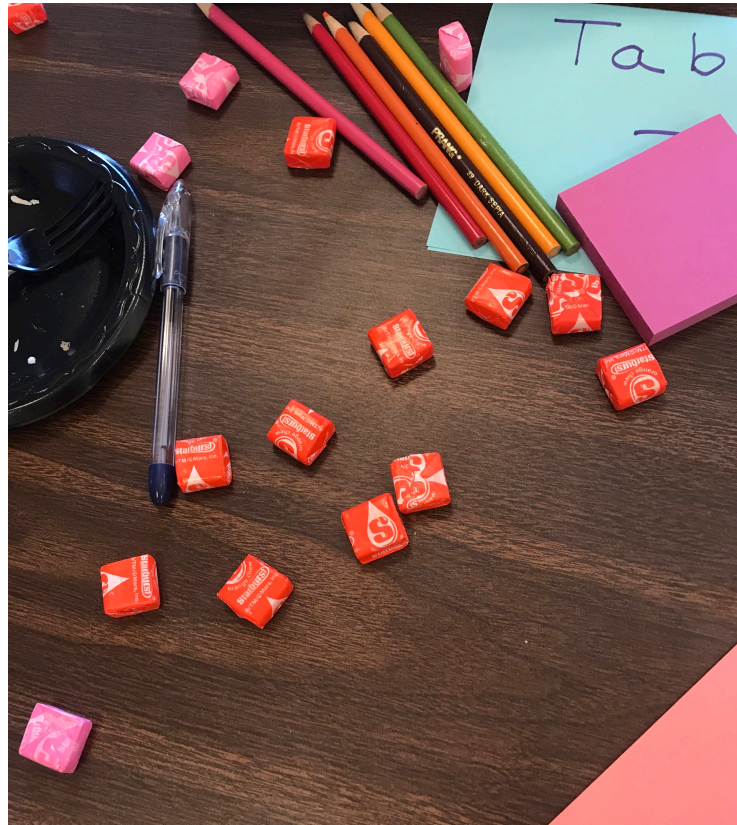
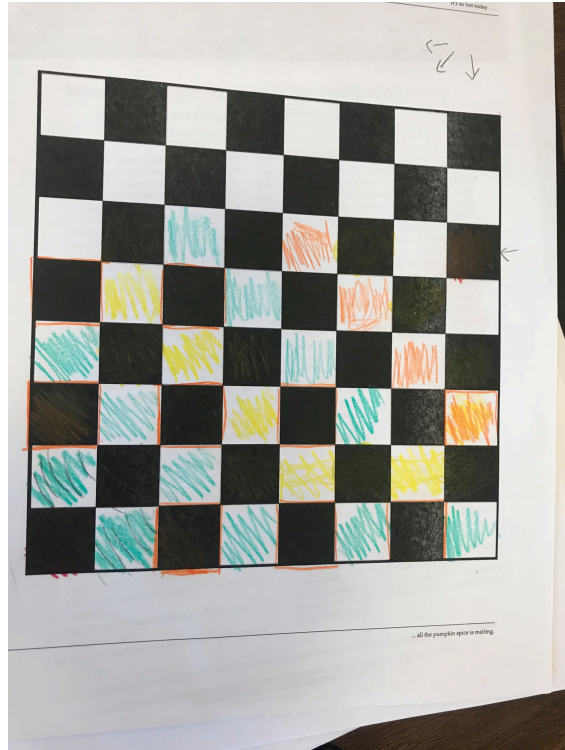
L	w	w	w
w	L	w	w
w	w	L	w

w	L	w	w
L	w	w	w
w	w	w	

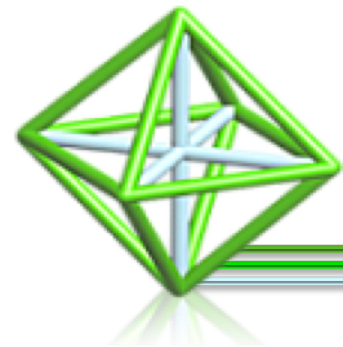
w	w	L	w
w	w	w	
L	w	w	w

w	w	w	L
w	w		w
w		w	w

Participant Work



Denver Participants



Chess Time

What do you notice? What questions do you have?

- The chess and flag problems were related. The types of flags said went with directions of chess pieces.
- Question: Does every chess question have a parallel flag question?
- The more freedom a piece had to move, the less choice you had to force a winning move.
- The starting position must have something to do with it. Starting on a point (e.g. $(8,5)$), which are relatively prime must have something to do with it.
- Is there a relationship between the starting position and the starting number of flags in each color (when the questions are related)? And what is that relationship?



Pythagorean Triples

- I didn't understand the Eisenstein triples.
- There was some relationships with the mod 4s. All the PPT were a combo of 3, 0, and 1. Hypotenuse was always 1 in mod 4.
- The multiple of 3, 4, and 5 is in every PPT. (not necessarily distinct numbers).
- The hypotenuse cannot be a multiple of 3 or 4. And must be odd.
- If the short leg is even, then the other leg has to be odd (and vice versa).
- What Becky said...

