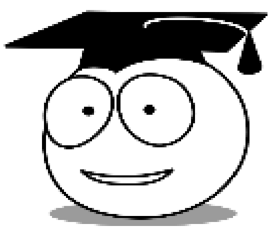
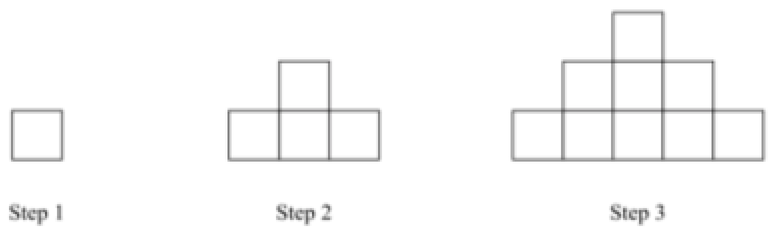
Make it Work, Part 1: Explore the mathematics of a rich task

Student Hat: Play with the mathematics. Ask, "What more can I explore?"

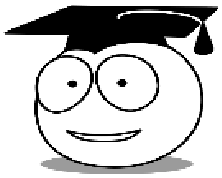
Teacher Hat: Consider how you, as a teacher, would use this problem to promote and record student thinking about mathematics.



(10 min) Activity 1a: For your first challenge:



As the step changes, __________ also changes.



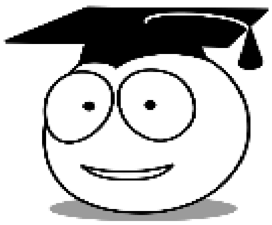
(25 min) Activity 1b: Do the Math!
With a partner, pick one attribute in our list and investigate. Make a conjecture and try to justify/ prove it.

Use whatever is convincing (graphs, tables, algebra, arguments).
If time, explore a 2nd or 3rd property

Share your work on poster paper. It can be messy!

Deadline: poster up by ______________________________

scratch work here



(20 min) Activity 1c: Gallery Walk (Get your Post-its!)
Explore the mathematics others saw in their problem.
Read and consider their arguments. Give Feedback.

(Post-its) consider the following prompts:
"I like that you …" (mention something specific about their work)

"I wonder if…" (mention something specific you are wondering about)"

"Next Step:" (mention something the creator might do next to move forward)

… Then read your feedback when you get back to your poster.

Activity 1d: (15 min) Exploring the mathematics in your work

part i: What mathematical **topics** emerged in your work, and the work of others?

part ii: What mathematical **practices** are demonstrated in doing this work? See the handout of the CCSS mathematical practices.

Activity 1e: (20 min) Preparing to implement a rich task with purpose.

In the second part, you will use this problem to help student learn something mathematically important. You will:

- Determine learning intentions and criteria for success.
- Plan questions, tasks and checkpoints to assess how well students are doing (relative to your goals).
- Prepare ways to give effective feedback that moves all students forward.
- Prepare ways for students to help themselves and their classmates more towards your learning intentions.

Group together as a TABLE:

i – Teams at each table should quickly summarize what you explored, what you learned, and what important mathematical topics and practices emerged. (3 min max for each mini-group)

ii - After your discussion, start discussing:

- one primary mathematical topic / content goal you want to design around in part 2.
- one primary mathematical practice you want to design around in part 2.

Make it Work, Part 2: Preparing to Implement a Rich Task

Non-negotiables describing Effective Assessment for Learning

- Clarify and share learning intentions and criteria for success with students.
- Engineer effective classroom discussions, questions, and learning tasks.
- Provide feedback that moves students forward.
- Activate students as the owners of their own learning.
- Encourage students to be instructional resources for one another.

Leahy, et al., *Classroom Assessment, Minute by Minute, Day by Day*. *Educational Leadership:* November 2005, Volume 63, Number 3. URL: http://bit.ly/Leahyarticle5nns

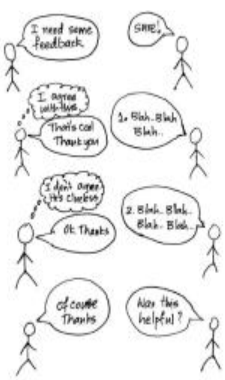
Activity 2a: (40 minutes) Now, your table will prepare to use this task as a way for your students to grow mathematically.

On Poster Paper: you will determine:

1. Learning goals/Intentions (*<u>many</u> are possible)*: *one* mathematical content goal and *one* mathematical practice that will *focus your decisions.*

2. Evidence of Success (Criteria): What does it *look* like in student work? What does it *sound* like in student comments? What do possible errors / misconceptions look / sound like?

3. Questions/Checkpoints: Plan at *one or two* questions/checkpoints that will give you good feedback about student progress on your goals. *Why will thesework?*



4. Feedback: How will students receive feedback about their progress? *Who will deliver the feedback? How can it be delivered?*

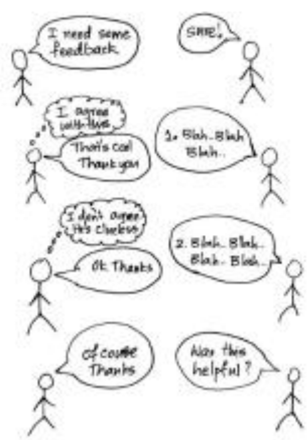
Deadline: Posters up by ________________________

Goals

Evidence of success

Checkpoints

Feedback



Activity 2b: Giving Critical Feedback

(20 minutes) Choose a poster, and give yourself time to truly understand their goals, evidence for success, questions/checkpoints, and plans for feedback.

Post-its: Give Specific Feedback on the *linkage* between:
Their goals ←→ *Their decisions for Evidence, Questions, Feedback*

Activity 2c: Receiving Feedback / reflection in your group, sharing:

(15 minutes) Read the feedback you received about your work.

Reflect: What did you learn after giving/ receiving feedback?

- Did you have clear learning goals?
- Is there more mathematics in this situation than you anticipated?
- Could you modify your decisions to help students reach goals?

Activity 2d: Exit Task

(5 min) On index card: How might you change one thing about your current approach to at least one task you use in the classroom? Be specific.

Resources:

http://www.visualpatterns.org

*Leahy, et al., Classroom Assessment, Minute by Minute, Day by Day. Educational Leadership: November 2005 | Volume **63** | Number **3***

Original URL: http://www.ascd.org/publications/educational-leadership/nov05/vol63/num03/Classroom-Assessment@-Minute-by-Minute,-Day-by-Day.aspx

URL: http://bit.ly/Leahyarticle5nns

Common Core State Standards: Mathematical Practices

The eight core practices that students should understand and enact in doing and thinking about mathematics:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

In particular:

1. Make sense of problems and persevere in solving them

- consider analogous problems, special cases and simpler forms
- transform algebraic expressions or change viewing window to obtain information needed
- use concrete objects or pictures to help solve a problem
- explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends
- make conjectures about the form and meaning of the solution and plan a solution path rather than jumping into a solution
- check answers to problems using a different method
- monitor and evaluate progress and change course if necessary
- understand and compare different approaches

2. Reason abstractly and quantitatively

- represent a given situation symbolically and manipulate the representing symbols
- stop and think about what the symbols represent in the context
- reason with quantities and about relations among quantities
- consider the units involved
- attend to the meaning of quantities, not just how to compute them
- knowing and flexibly using different properties of operations and objects

3. Construct viable arguments & critique the reasoning of others

- make conjectures and build a logical progression of ideas
- use stated assumptions, definitions and previously established results in constructing arguments
- determine domains to which an argument applies
- analyze situations by breaking them into cases
- recognize and use counterexamples
- compare effectiveness of two plausible arguments
- distinguish correct reasoning from that which is flawed and explain any flaws
- justify conclusions and communicate them to others

4. Model with mathematics

- write an equation to describe a situation
- apply mathematics to solve problems arising in everyday life, society, and the workplace
- identify important quantities in a practical situation and map their relationships using diagrams, two-way tables, graphs, flowcharts and formulas
- make assumptions and approximations to simplify a complicated situation
- interpret mathematical results in the context of the situation and reflect on whether the results make sense, improving the model as necessary

5. Use appropriate tools strategically

- make sound decisions about using tools, recognizing both the insight to be gained and their limitations
- use technology to visualize the results of varying assumptions, explore consequences, and compare predictions with data
- use technological tools to explore and deepen understanding of concepts
- identify relevant external mathematical resources and use them to pose or solve problems
- analyze graphs, functions and solutions generated by technology
- detect possible errors by using estimation and other mathematical knowledge

6. Attend to precision

- communicate precisely to others
- use clear definitions in discussion and in reasoning
- state the meaning of symbols used, specifying units of measure, and labeling axes
- calculate accurately and efficiently
- note the assumptions made
- express answers with an appropriate degree of precision

7. Look for and make use of structure

- look closely to discern a pattern or structure
- see complicated things (such as algebraic expressions or functions or a histogram) as single objects or being composed of several objects
- recognize and use the strategy of drawing auxiliary lines to support an argument

8. Look for regularity in repeated reasoning

- notice if calculations are repeated
- look for general methods and for shortcuts
- evaluate the reasonableness of intermediate results

A Case Study: Making a Rich task work

My Experience with the "trains problem:" what I prepared, what I saw, what I learned.

The problem:
Wendy has "cars" of length 1 and 2. She will attach them together to make trains of length 5. How many different trains can she make? A 2-1-2 train is different from a 2-2-1 train. How many different cars can she make of length 6? length 8? length *n*?

Important Mathematics in the Task:

- Generating a systematic counting strategy
- Presence of recursive relationships, possible match to the Fibonacci Sequence
- Relationship of "trains" task with the Fibonacci numbers
- Expressing the Fibonacci numbers non-recursively using combinatorics
- Proving the general case with proof via mathematical induction

One way to express a solution:

$$T(n)=\binom{n}{0}+\binom{n-1}{1}+\binom{n-2}{2}+\cdots+\binom{n-\lfloor n/2\rfloor}{\lfloor n/2\rfloor}, \text{ or } \sum_{i}^{\lfloor n/2\rfloor}\binom{n-i}{i}$$

Important Mathematical Practices / Habits of Mind:

- Make sense of problems and persevere in solving them
    - Launch of the problem: clarifying what the task asks for,
- Reason abstractly and quantitatively
    - Organizing work into groups,
- **Construct viable arguments and critique the reasoning of others****
    - Verifying work for trains of length 5 or 6,
    - Group setting, understanding/listening to student arguments in whole-class discussion
- Model with mathematics
    - Representing train lengths with notation, formulas
- Use appropriate tools strategically
    - Systematic counting strategies,
    - Applying a proof by induction in this setting
- Attend to precision
    - Checking details in lists, did I repeat a train
    - Being precise / clear about notation in formulas, “mathematizing” their work
- Look for and make use of structure
    - Seeing patterns in their list, categorizing,
    - Classifying all train of size 5 or 6 into groups
- Look for and express regularity in repeated reasoning
    - In a general proof, taking the results from $n = 5$ and $n = 6$ to justify their solution for $n = 7$

Goals for my students:
Math goals:

- ***Students will construct viable arguments and critique the reasoning of others' arguments.***

- Students **will be able to conjecture a formula for a general case**, based on observing a series of specific examples.

- Students will recognize that the **detection of a pattern is not equivalent to a mathematical proof.**

- Students will better understand how the technique of mathematical induction "works" as a proof technique for the general case.

What I looked for in Student Work (common misconceptions /errors):

- Will they recognize that formulas emerge from specific cases, or will they resist the specific case and go for the general case, because “there’s always a formula?” (Evidence: “waiting for the formula”)
- Will they attach algebraic expressions to undefined or inappropriate quantities? (“You do the T sub n thing here, because that’s what you do!”
- Will their method of counting be exhaustive / systematic? Can they guarantee no repetitions or over-count? (“I got ‘em)
- Will “proof by consensus” be a substitute for valid verification (We all got ‘em”)?
- Will they not recognize a need for a proof (“It’s Fibonacci! We’re done! Yay!”)?

Questions /checkpoints I prepared in advance:
Launch of the task/statement of problem:

- Start with 5 and 6 before the general case…. See if they jump to the general case without valuing the specific cases.
- Start with 5 and 6, not 1, 2, or 3: *simpler cases not complex enough to expose the complexity of the task. May also motivate looking at simpler cases.*
- Deliberately skip 7. *Finding all trains of length 8 is long and irritating. Motivate the need to generalize, use previous work.*

Walking around the student groups:

- How do you know you are done?
- How are you organizing the trains in your list?
- How do you know that you have no overlaps or repeats?

Once students started getting answers for 6: **Whole group**

- What methods are students using to classify all trains of size 6?

Once most students start on *n = 8*:

- How is the case for trains of length 8 going? Having fun yet?
- I wonder if there's an easier way to get the answer for trains of length 8.

Decisions about orchestrating the task

- **Silent Individual read / think time** to prevent "speed demons" from sabotaging the task for more deliberate, thoughtful students.
- **Group Time:** To talk out ideas, look at other approaches, critique the reasoning of others.
- **Whole-class time:** to clarify task language, intention,

- **Random assignment of groups** to make all students responsible for learning from / contributing to the progress on the task
- **Collecting data and crouching:** I am tall and big, and kids get scared of me when I approach a group. Hard for me to "stay out of the way" while listening to students work with each other. They think they are supposed to talk to me.

What I saw monitoring student groups:

- Some of the "most accomplished students" bristled at the "baby work" of doing simpler cases, and their struggles to connect the specific to the general surfaced. This threw them off the "top of the mountain."
- Nearly all students generated complete list of trains of size 5 and 6.
- Students initially struggled to see how to organize the elements of their list in a way that would facilitate insight into counting the general case. There was progress, but not for every student.
- Students were regularly attending to precision in their work – checking with others, making sure they didn't repeat / over-count.
- Some students did not value / see the value in the work of other student ideas. They see it as either cheating, or giving the other students an advantage.
- Some students de-valued and abandoned their *own* ideas after seeing different approaches from others deemed "smarter." They did not truly evaluate / deeply understand the work of others.

What I noticed after giving them their homework task to sketch out the structure of a "proof" by induction

- Students did things other than what I had asked.
- Students did not understand the question.
- Students did not understand that this proof by induction requires that *two* initial cases need to be verified.
- Students could not apply a proof by induction in this setting.

What I will do differently next time:

- Propose a different "exit" task / question: what do you get? What questions do you have?
- Give this task at the *beginning* of their work on sequences, sums and proofs by induction. Re-visit the task throughout the chapter, with different goals / lenses to focus on.