

# PCMI November Outreach Weekend

12-13 November 2016

Math!



# Survivor Time - Morning

## What did you notice about the 21-Flags Survivor Game?

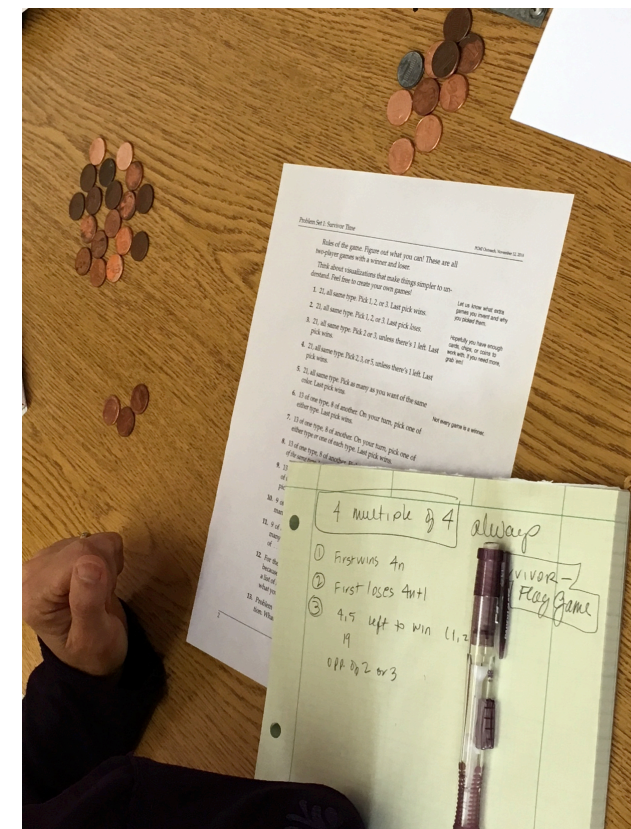
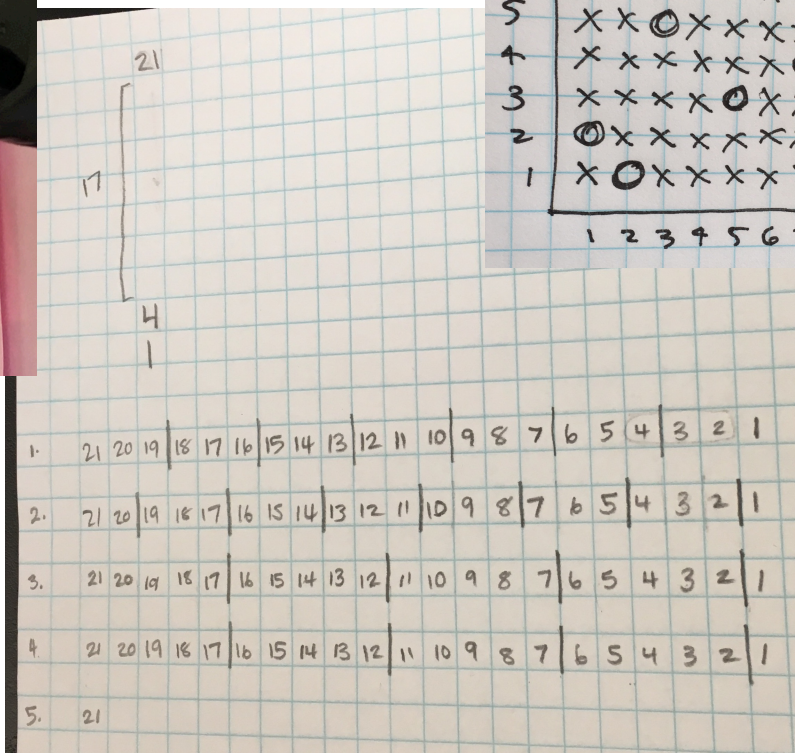
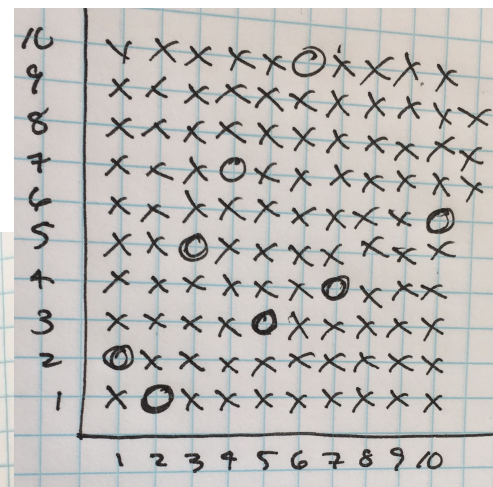
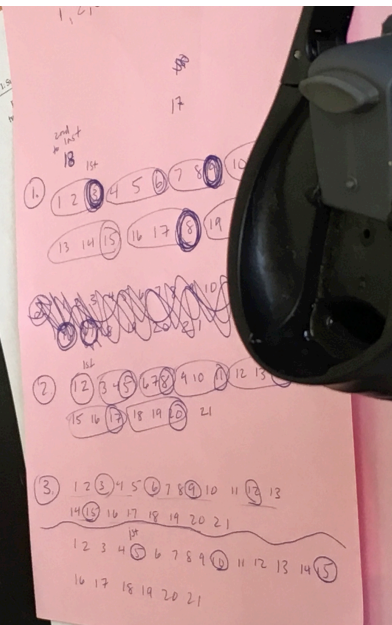
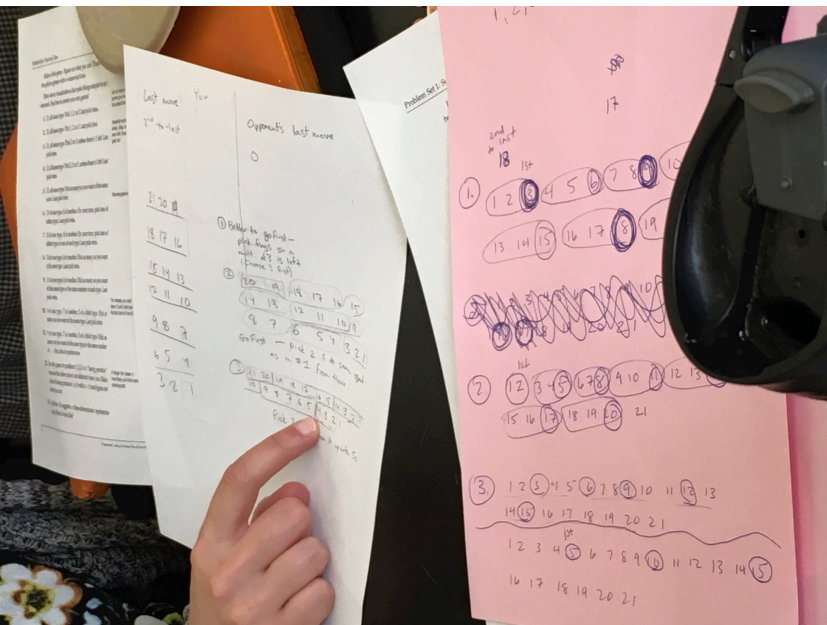
- Little time given on Survivor. Didn't notice until end that they were "screwed."
- Grouping pennies was useful.
- Multiples of 4 bad. Seems to be true for Game 4 too.

## What about the other games? (Specifically games 2-8)

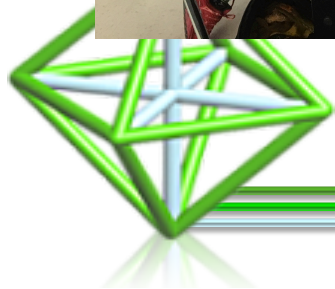
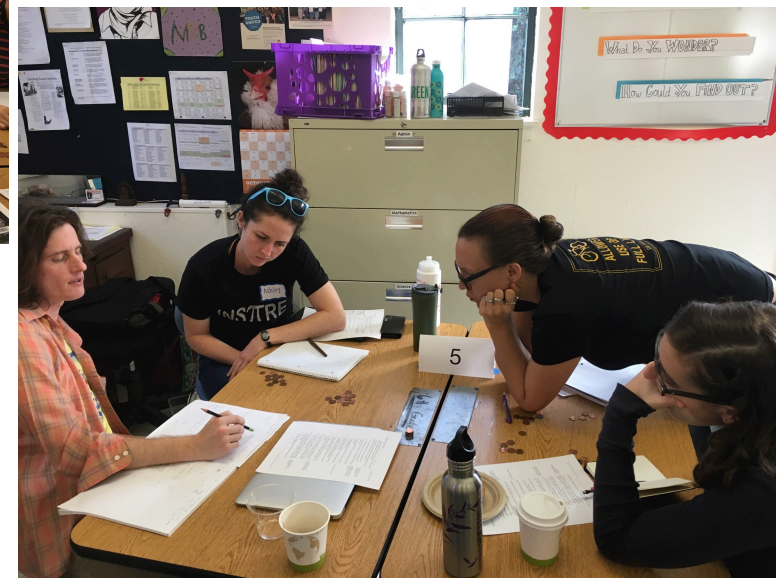
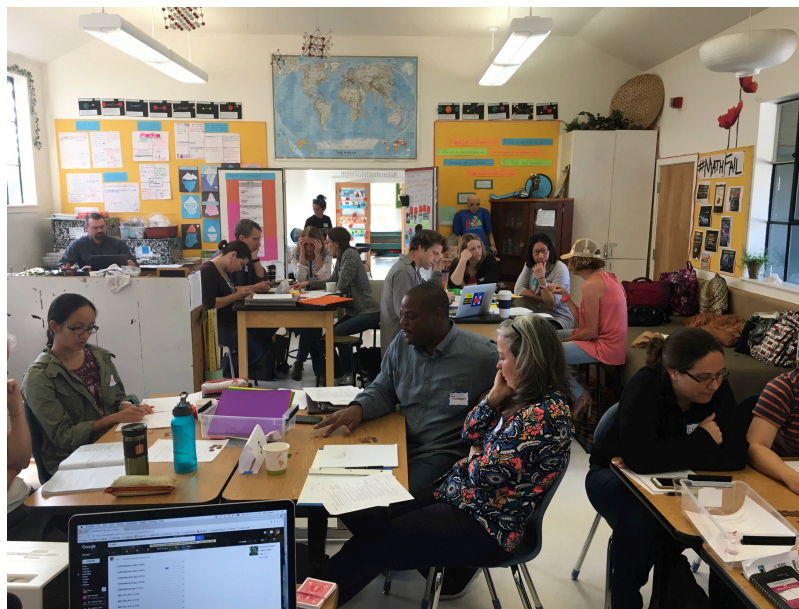
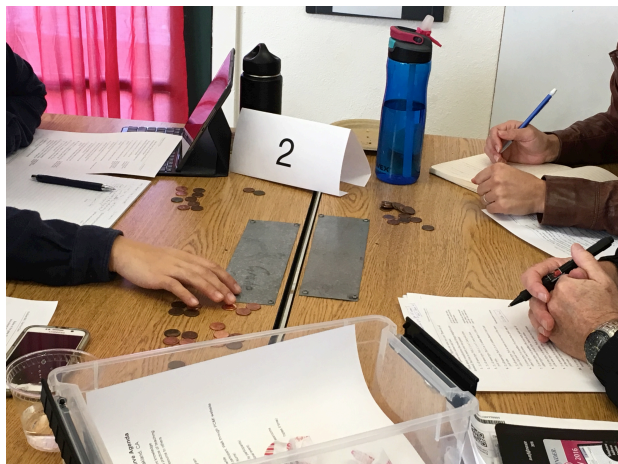
- In General: What was the winning strategy at the end? How to win from there... Working backwards.
- Looked at different combos by grouping (Game #3) to see what would win. What would wipe the other out?
- Used a grid. Looked at 2 variables (coordinate plane?).
- Correct language was important for discussing the scenarios – are we talking about leaving team A with X or team B? Be clear.
- Looked at groupings and combos that were winning/losing scenarios.



# Participant Work



# Morning Math!



# Survivor Time - Afternoon

## What did you notice about the other games?

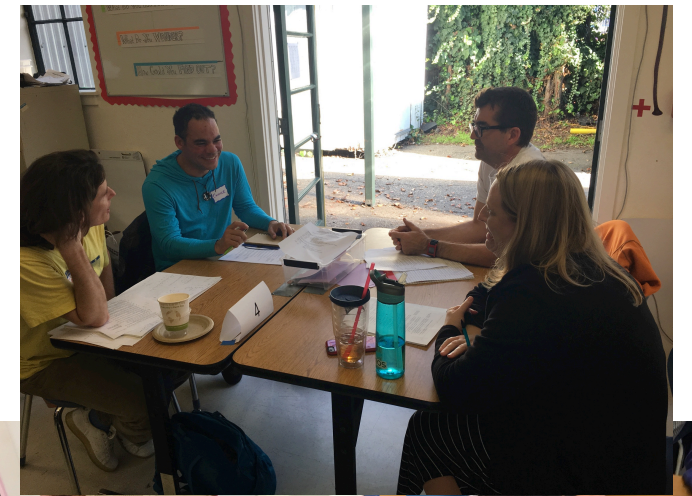
- Game 7: Strategy of maintaining even-even in both piles.
- Experimented with the coordinate plane – still working through to see what combos win. Got to the idea of one even number helping (Game 7 as well).

## What questions do you have? Wonderings?

- Can someone explain how the 3D representation works (For #13).
- Seems like there is a favor to being either first or second player. Doesn't seem to be a game where it doesn't matter if you go first or second.
- Noticed that the beginning game position for #9 was the losing position. And we couldn't solve #9 until we did #12.
- If you leave the piles with 1 more than the other one on #9, then you can win.
- Could turn #10 into #8. (strategies of tying other games together)
- Is there a significance here with Fibonacci numbers?
- #12: Wythoff Sequence?



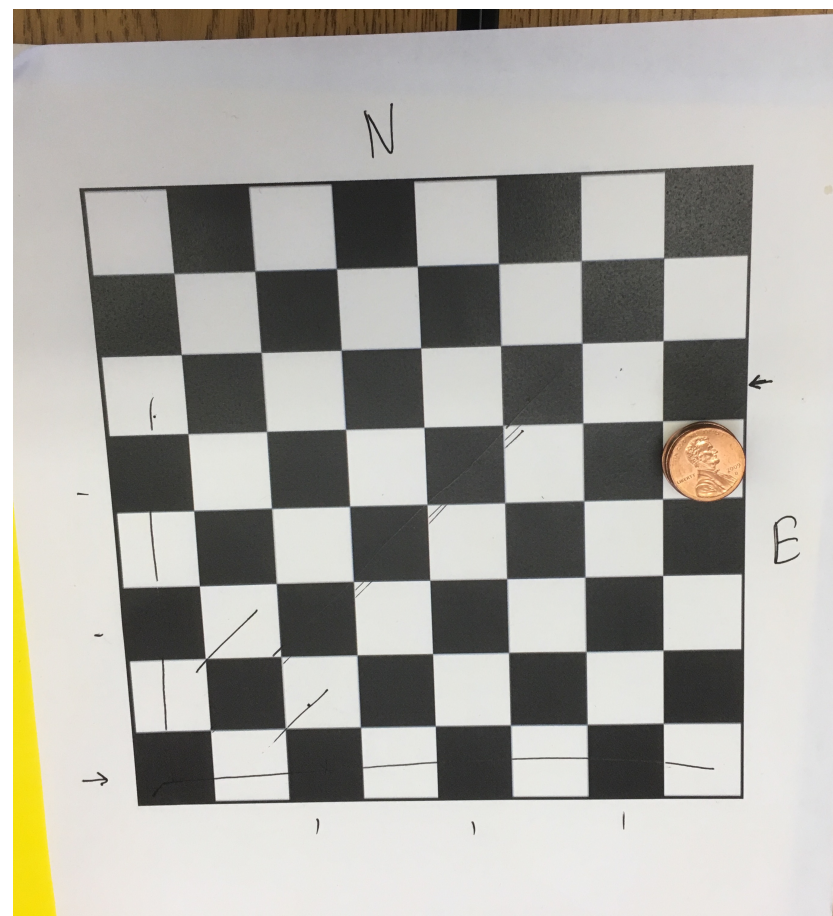
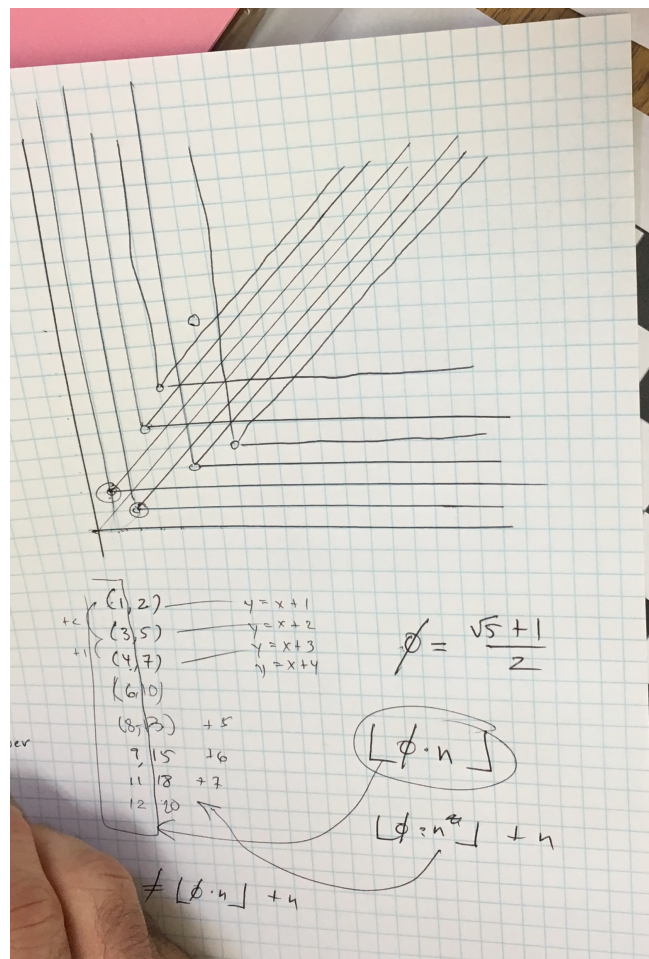
# Afternoon Math!





# Oakland Math!

# Oakland Math!



#7 on p51 who puts the king in the corner. The one who puts the king in the corner is the winner. If both players play with perfect strategy, who wins, and how? Oh, you'll also need to figure out what perfect strategy means.

#8 on p51 A chess rook (castle) sits three squares below the top right corner of a chessboard. Same rules. Who wins, and how?

#9 on p51 A chess queen sits three squares below the top right corner. Same rules. Who wins, and how?

Bring your new knowledge to bear on some of the problems you did yesterday. What do you notice?

Repeat the king and rook problems. Oh wait, now these pieces are in 3-D, and they start 7 units to the east, 4 units to the north, and 8 units up from the target space.

Repeat the queen problem in fabulous 3-D. You'll have to decide how a queen moves in 3-D.

Strategy for #7  
 $X = \text{my move (I go 1st)}$   
 $O = \text{opponent's move}$

# of flags of type 2 left

# of flags of type 1 left

2

$X = 2^{\text{nd}} \text{ person}$   
 Always go to diagonal

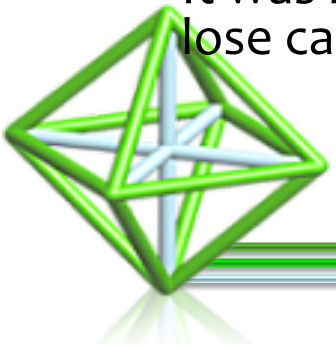
If you can't go, you lose



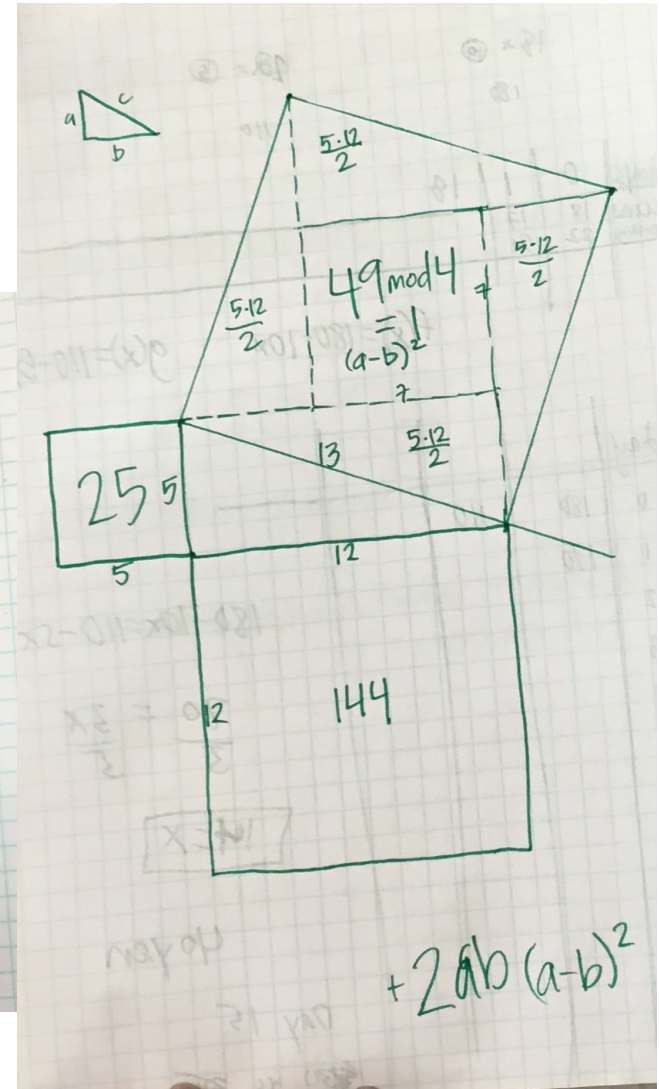
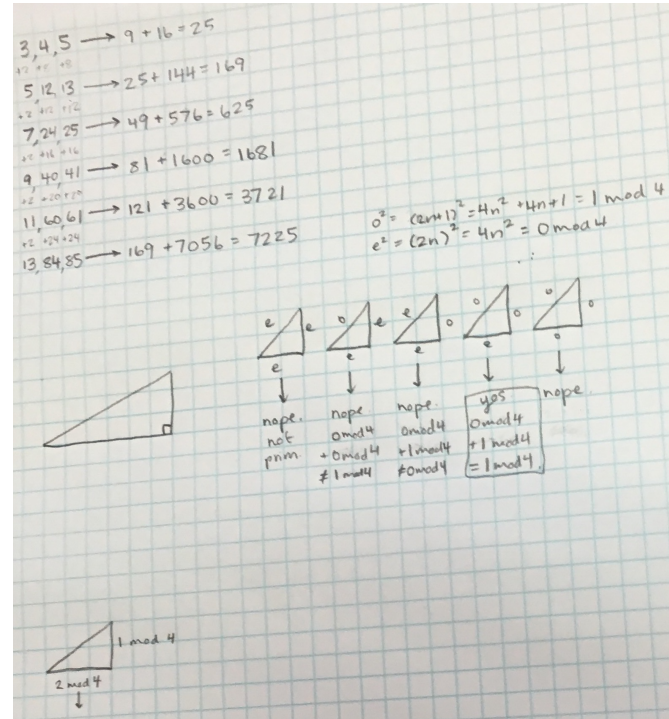
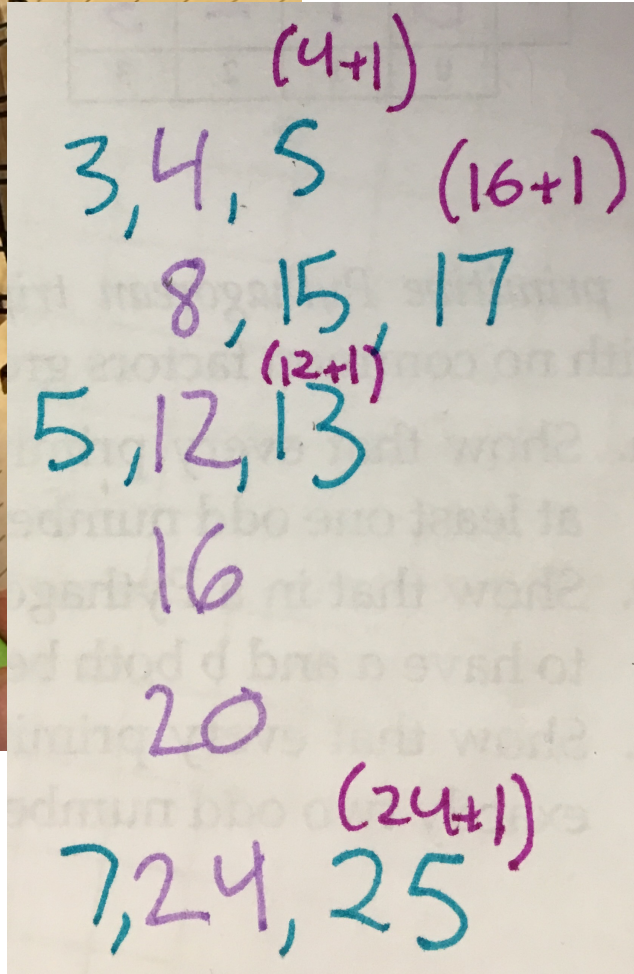
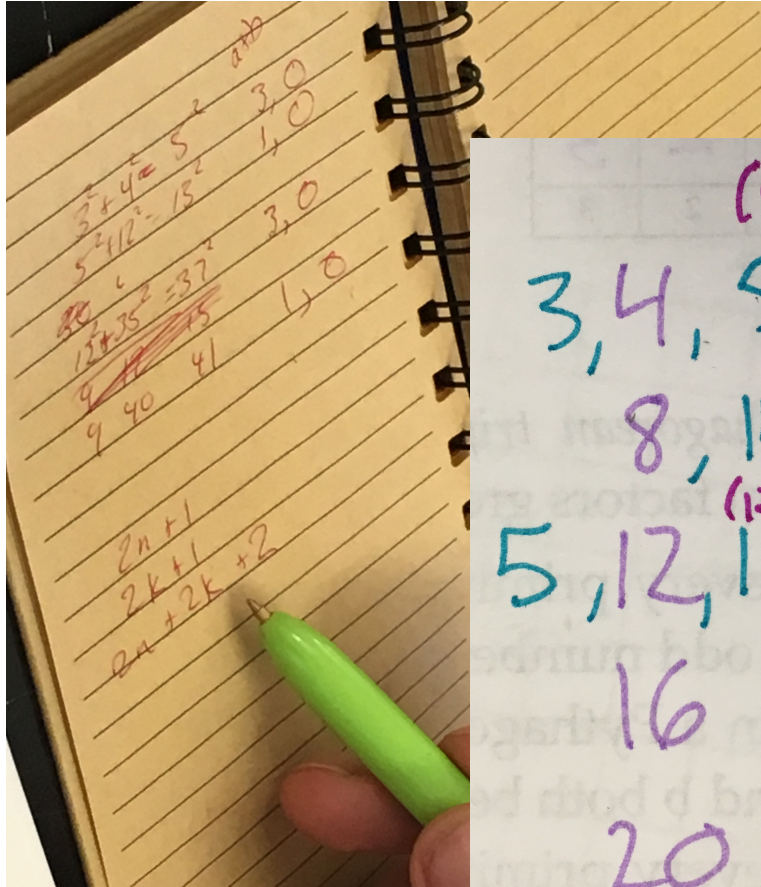
# Chess Time

## What questions do you have? Wonderings? Noticings?

- It was difficult to play with an 8 by 8 by 8 chessboard, so we stacked pennies.
- #7: Connected to Fibonacci. First player will always win.
- Have a better understanding of how pieces move in 3D. 3D was a big challenge – but using the 3 different types of tokens helped visualize the movement along the diagonals.
- How can we use the protocols for group shuffling in our own classrooms for effectiveness. Hearing about stacking the pennies from another group was cool!
- How to extend 2D strategies to 3D. Low confidence in some really neat solutions. What does diagonal mean in 3D? The colors on different levels do not match for all diagonals. Controversy!
- Reviewed 2D strategies. How would this sort of problem set look in our classrooms? How would students work in groups on these problems? Creating a climate where all are involved.
- It was helpful to go back to the physical aspect of moving pieces in order to see relationships and lose cases. Still searching for answers! Better understand movement of pieces.



# Pythagorean Triples!



# Pythagorean Triples

## What questions do you have? Wonderings? Noticings?

- There's always a multiple of 4 in a Pythagorean Triple, and the hypotenuse,  $H$ , is  $(H) \bmod 4 = 1$ .
- The sum of  $a$  and  $b$  is always 3 or 1 mod 4, and the product  $ab$  is always 0 mod 4.
- Classified PTs with lowest # being odd vs even. There were some sides that were 2 apart. Sides as  $a^2 + b^2$ ,  $a^2 - b^2$ ,  $2ab$ . What happens when  $a$  and  $b$  are 1 apart, 2 apart, etc.
- Tried to connect Q1 and Q2. mod 4 is mod  $2^2$
- Q3: Every PT has to have a multiple of 2, 3, and 5. (but can be in the same number). All have 2, 3, 4, and 5? e.g. 5, 12, 13 (12 is a multiple of 2 and 3, 5 is a multiple of 5)
- Challenge: proving via argument vs many examples.  
 $(3n)^2 = (3n) \bmod 4$
- Discussed algebraic formula and used multiplication chart to find PTs. Make visual (see pic to the right) ->

Printable Multiplication Table 20X20

Handwritten notes above the table:

- $x^2 = 9$ ,  $x = 3$
- $y^2 = 25$ ,  $y = 5$
- $2 \times y = 30$ ,  $x \times y = 15$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289

